CREATE

Centre de Recherche en économie de l'Environnement, de l'Agroalimentaire, des Transports et de l'Énergie

Center for Research on the economics of the Environment, Agri-food, Transports and Energy

When Hotelling meets Vickrey: Service timing and spatial asymmetry in the airline industry

André de Palma

Carlos Ordás Criado

Laingo M. Randrianarisoa

Cahier de recherche/Working Paper **2017-1**

Janvier/January 2017

de Palma : Professor at the Department of Economics and Management, École Normale Supérieure de Cachan, Paris, France.
Ordás Criado: Professor at the Department of Economics, Université Laval and CREATE, carlos.ordas@ecn.ulaval.ca
Randrianarisoa: Postdoctoral Fellow at Sauder School of Business, University of British Columbia, Vancouver, BC, Canada.

ISSN 1927-5544

When Hotelling meets Vickrey

Service timing and spatial asymmetry in the airline industry*

André de Palma[†] Carlos Ordás Criado[‡] L. M. Randrianarisoa[§]

Abstract

This paper analyzes rivalry between transport facilities in a model that includes two sources of horizontal differentiation: geographical space and departure time. We explore how both sources influence facility fees and the price of the service offered by downstream carriers. Travellers' costs include a fare, a transportation cost to the facility and a schedule delay cost, which captures the monetary cost of departing earlier or later than desired. One carrier operates at each facility and schedules a single departure time. The interactions in the facility-carrier model are represented as a sequential three-stage game in fees, times and fares with simultaneous choices at each stage. We find that duopolistic competition leads to an identical departure time across carriers when their operational cost does not vary with the time of day, but generally leads to distinct service times when this cost is time dependent. When a facility possesses a location advantage, it can set a higher fee and its downstream carrier can charge a higher fare. Departure time differentiation allows the facilities and their carrier to compete along an additional differentiation dimension that can reduce or strengthen the advantage in location. By incorporating the downstream carriers into the analysis, we also find that a higher per passenger commercial revenue at *one* facility induces a lower fee charged by *both* facilities to their carrier and a lower fare charged by *both* carriers at their departure facility, while a lower marginal operational cost for *one* carrier implies a higher fee at its departure facility, a lower fee at the other facility served by the rival carrier and a lower fare at *both* facilities.

JEL Classification: D43, L13, L22, L93, R4

Keywords: Airline and facility competition, Horizontal differentiation, Location model, Spatial asymmetry, Service timing.

---

*This version: January 24, 2017.

[†]École Normale Supérieure de Cachan, Paris, France.

[‡]Corresponding author: carlos.ordas@ecn.ulaval.ca, Department of Economics, Université Laval, Québec, QC, Canada.

[§]Sauder School of Business, University of British Columbia, Vancouver, BC, Canada.

1 Introduction

Since the deregulation of the airline market in the US in the mid '70s and in Europe in the '90s, rivalry between airports and their carriers has intensified. Travellers located in populous regions often have the opportunity to choose between multiple airports for a given destination. While considerable theoretical literature is devoted to analyzing rivalry in prices and service frequency across facilities supplying perfect substitutes, a fundamental aspect of the air travel market has received less attention: departure time competition. The convenience of schedules is of great concern for travellers. For carriers, service timing provides an additional differentiation dimension that can be strategic for competition. Transportation authorities stand to benefit from improved understanding of the role played by service scheduling in deregulated markets.

We propose a facility-carrier model to study how facility fees, carrier fares and departure times are set when geography gives more market power to one of the facilities and their carriers. Hotelling (1929) has been widely used in industrial organization to study spatial competition and the level of product diversity provided by an imperfectly competitive market. Concurrently, since Vickrey (1969), time costs have become fundamental components to model firms' and consumers' timing decisions (travel, departure or arrival times). Consumers' value of time and schedule delays, defined as the difference between the desired and the actual departure/arrival/total time, are widely used for modelling congestible infrastructures. The costs related to the timing of the service is also central in carriers' planning. Our paper appears to be the first to attempt bringing these two frameworks together, by considering spatial differentiation and a location game in departure times into the same model in order to analyze consumer behaviour and the pricing and schedule decisions of rival facilities and their carriers.

Research explicitly recognizes that the airline market operates under different forms of imperfect competition and that pricing the access to the facilities (through a congestion toll or slot[1] management) is an efficient way to address congestion.[2] De Palma and Leruth (1989) and De Borger and Van Dender (2006) investigate the capacity and price decisions of congestible facilities selling perfect substitutes in duopolistic markets by using sequential capacity-price games. Van Dender (2005) explores how duopolistic providers of perfect substitutes with fixed capacities set prices when access to each facility is subject to road congestion. While the aforementioned studies consider the facilities as final service providers, Basso and Zhang (2007) analyze rivalry between congestible facilities in "a vertical structure". They consider the facilities as input providers (upstream firms) that reach final consumers only through carriers (downstream firms). In their setup, carriers may possess market power in the output market,

[1] A slot is defined as the entitlement to use a runway at a particular time.

[2] See De Borger et al. (2007); De Borger and Proost (2012) regarding the strategic pricing and capacity decisions of governments in congested infrastructures, and Zhang and Czerny (2012) for a recent literature review focusing on airports and airlines.

and the characteristics of this market affect the pricing and capacity decisions of facilities. They show that duopoly facilities have lower fees than the monopolist but lower frequencies (service quality), depending on the timing of the fee-capacity decisions. They also compare the capacity and service frequency decisions of the monopolist with those of the social planner and find that, conditional on facility fees, the optimal decisions coincide only if carriers operate in perfectly competitive markets. Brueckner (2002, 2009) and Pels and Verhoef (2004) extend the theory of congestion pricing developed for road traffic to congested airports[3] when the final service providers − the carriers who want to depart/land at peak hours − have market power. Verhoef (2010) looks into alternative instruments (such as slots sales and slots trading). Thus, understanding carriers' scheduling decisions requires taking into account their time-related operational costs, which we do.

Since Vickrey (1969), consumers' valuation of time has become under more intense scrutiny in theoretical and applied transportation economics work. When pertaining to air travel, the pioneering papers are Douglas and Miller (1974) and Panzar (1979). Panzar proposes a spatial model with free entry in which two profit-maximizing airlines each operate a single flight and consumers' generalized costs depend on fares, flight frequency and schedule delay costs.[4] The analysis focuses on fares and frequency equilibria but ignores departure time competition. A few papers explicitly incorporate departure time rivalry into the analysis. Encaoua et al. (1996) are the first to consider a time-then-fare game in a carrier network involving two direct and one indirect connection between three cities. Assuming uniform desired departure times for the travellers over the time of day and quadratic schedule delay costs, various Nash equilibria in fares and departure times are characterized and minimum differentiation in schedules is established in selected configurations. Under the same assumptions regarding travellers' schedule delay costs, Lindsey and Tomaszewska (1999) consider a sequential model where multiple service times are chosen before fares on a city pair route served by two airlines. A multinomial random utility model captures the utility of travellers across alternatives. Predatory behaviours, in which an airline attempts to hurt (potential) rivals, are also investigated. They show that predatory fare cutting is less effective than a predatory schedule but the outcome depends on the prey's schedule response. Their results rely on numerical simulations. More recently, Van der Weijde et al. (2014) investigate the schedule decisions of two duopolistic travel operators by using a horizontal differentiation model with price-sensitive demands and asymmetric (piecewise linear) schedule delay costs. Departure times are treated as locations on a fixed time interval and each operator schedules a single time of departure. A thorough

[3]The main costs associated with airport congestion are: increased access time/cost for travellers, slots shortage, take-off queues, landing delays for the carriers (which, in turn, impose inconvenience to travellers).

[4]Schedule delay costs designate the monetary costs associated to departing or arriving earlier/later than desired.

analysis of time-fare games with separated and covered markets is proposed. Assuming uniform desired times for consumers, they show that the simultaneous game has no equilibrium. Only if one operator sets its fare and departure time before the other can an equilibrium exists. The time-then-fare game also has a stable equilibrium in fares and times, and results in services scheduled closer than socially optimal but not necessarily in minimum differentiation. As the authors state, one drawback of their analysis is that most equilibrium expressions have no intuitive interpretation.[5]

A number of researchers[6] have evaluated passengers' value of time in air transportation empirically, relative to other characteristics of the trip. They all confirm that travel demand is linked to the timing of the service (arrival or departure time) or to total travel time, in addition to the cost/time to access the departure facility and to other characteristics of the trip. The principle of minimum differentiation[7] in departure times has been empirically tested in commercial aviation for the US and Norwegian markets by Borenstein and Netz (1999) and Salvanes et al. (2005) before and after airline deregulation. Both researches conclude that time differentiation was reduced after deregulation.

Our work is in the vein of the spatial approaches. It borrows "the vertical structure" proposed by Basso and Zhang (2007) in a spatial setting but drops the congestion components to include schedule decisions in the spirit of Van der Weijde et al. (2014). Another distinguishing feature of our model is that it posits the existence of a spatial asymmetry in the location of one of the facilities. Closer proximity of travellers to a primary facility (and to their carriers) typically results in a spatial advantage with respect to a rival secondary facility settled in a remote place, due to the transportation costs incurred by travellers. Similarly, the costs/constraints faced by the carriers in the timing of the service may lead to different departure times, which may hurt a carrier (and its departure facility) if the schedule is less convenient to travellers. In commercial aviation, peak/off-peak charges and slots acquisition costs can be sizeable components of airlines' operational costs in congested airports.[8] Our model allows these time costs

[5]To circumvent the intractability of the spatial approaches in dealing with the timing of the service in theoretical models, Brueckner and Flores-Fillol (2007) and Brueckner (2010) propose to study scheduling competition through frequency competition between suppliers in fare-frequency games. They consider that individuals care for overall flight frequency rather than individual departure times. Higher frequencies are valued by passengers since they imply a broader range in choice of departure times. Thus, schedule delay costs are inversely related to frequencies.

[6]See, among others, Pels et al. (2000, 2003); Adler et al. (2005); Brey and Walker (2011).

[7]See De Palma et al. (1985) for a theoretical discussion of the minimum differentiation principle under heterogeneity in consumers' tastes. This work also considers a model with two types of horizontal differentiation: along a line and along a circle. The cylinder model was studied by Ben-Akiva et al. (1989).

[8]Worldwide airports are classified by the International Air Transport Association (IATA) into three categories. Level 1 and 2 airports, also called *non-coordinated and schedule facilitated airports*, designate facilities where capacity adequately meets demand and slots are freely set by airlines. Level 3 airports, also called *fully coordinated* airports, are those where demand exceeds capacity and the slot

to differ across downstream firms and to impact their fare and schedule decisions. In addition, while the previous literature considering departure time games assumes evenly distributed desired times for consumers throughout the time of day, our setup remains agnostic about the shape of this distribution. To keep the analysis tractable, we assume that only one carrier serves each facility at a single time. Transport facilities earn their revenue from services offered to their carriers and from rights granted to external firms to provide on-site services to travellers (often called commercial revenues). These commercial revenues can seriously impact facility charges.[9] Thus, we also account for them when we model facility decisions.

The interactions in the vertical facility-carrier model are represented by a three-stage game with simultaneous decisions at each stage. In the first stage, two profit-maximizing facilities located in a linear city set the per passenger fees they charge to their carrier and announce their operating hours. Then, knowing the fee charged at their facility, the downstream carriers choose their departure time. Given departure times, carriers set their fare in the last stage. Consumers decide whether or not to travel and if so, which facility they depart from. The game is solved by backward induction and results in a subgame perfect Nash equilibrium.

The paper is organized as follows. Section 2 solves the sequential game by starting from the end of the game. Section 2.1 characterizes consumers' demand for the final service. Section 2.2 examines the carrier rivalry subgame assuming either exogenous or endogenous service times and establishes carriers' equilibrium fare, departure time, demand and profit. Section 2.3 focuses on the facility-rivalry subgame and characterizes facilities' equilibrium fee, demand and profit. We analyze the socially optimal location of the facilities and departure times in Section 3. The last two sections provide simulation results and a summary of our main findings along with possible extensions.

2 The Model

In a linear city of unit length, potential consumers are uniformly distributed with a density of one consumer per unit of length. Two facilities ($i = 0, 1$) serve the city and a single carrier at each facility schedules a homogeneous service at time T_i during the operating hours of its departure facility. The opening and closing times of the facilities, denoted $\underline{T}, \overline{T} \in]0, 24[$, are exogenously set and such that $\underline{T} < \overline{T}$.[10] Facility 0 is located

allocation is resolved through the IATA Scheduling Process. Secondary airports are mainly classified into level 1 or 2.

[9] In airports, commercial revenues typically include retailing, advertising, car parking, car rentals and banking. The importance of non-aeronautical revenues (or concessions) in airport profitability have been studied by Zhang and Zhang (1997, 2003) and Oum et al. (2004). See Zhang and Czerny (2012) for a recent discussion on how concession revenues affect private and public airport behaviours.

[10] Opening and closing hours of the facilities play no major role in our analysis, but they are included to identify where they could affect our setup.

at point h, with $h \in [0, \overline{h}]$, $\overline{h} < 1$, and facility 1 is located at the end of the city at 1. The location of the facilities is given. In this setting, facility 0 and its carrier have a location advantage in the sense that, for equal prices, facility 0 faces a higher demand than its competitor. In what follows, we will mainly think of the service as being a trip or flight, carriers as airlines, facilities as airports and consumers as travellers.

2.1 Consumer choice

We assume that consumers select one airport and a flight on the basis of fare $\widehat{p}_i$, transportation costs and schedule delay costs that capture the monetary value of the inconvenience caused by departing earlier or later than desired. Consumers' desired departure times, denoted by t, are heterogeneous and distributed according to a strictly positive density $\rho(t)$ on the $[0, 24]$ time interval (referred to as the time line below). We denote by $F(t)$ the related cumulative distribution function (or CDF). Departure times are given to consumers and can differ across carriers. The total cost or "full fare" of the service for a potential consumer located at $x \in [0, 1]$, selecting facility i and with desired departure time t, is given by:

$$\widehat{p}_i + \widehat{C}(T_i, t) + \frac{\theta}{2} d_i^2(x),$$

where $\widehat{p}_i \geq 0$ is the fare at facility i and $\theta \in [\underline{\theta}, \overline{\theta}] \subset \mathbb{R}_+$ is the transportation cost per unit of *squared* distance, denoted $d_i^2(x)$, between consumer's location at x and facility i's location where $d_0^2 = (x - h)^2$ and $d_1^2 = (1 - x)^2$. From the consumers' perspective, a quadratic transportation cost is justified when the marginal disutility to access the transport facility increases with distance. This cost is also quite common to model firms that locate apart from a rival over a linear geographic market.[11] Term $\widehat{C}(T_i, t) \equiv \widehat{C}_i$ captures the schedule delay cost (in monetary units) incurred by a traveller for consuming the service offered at time T_i by the carrier operating at facility i. We assume that consumers' schedule delay cost function is piecewise linear in t:

$$\widehat{C}(T_i, t) = \widehat{\beta}(t - T_i)\mathbf{1}_{t \geq T_i} + \widehat{\gamma}(T_i - t)\mathbf{1}_{t < T_i},$$

where $\widehat{\beta} \in [\underline{\beta}, \overline{\beta}] \subset \mathbb{R}_+$ represents the unit cost of departing earlier than desired, $\widehat{\gamma} \in [\underline{\gamma}, \overline{\gamma}] \subset \mathbb{R}_+$ is its late counterpart and function $\mathbf{1}_A$ is an indicator function that equals 1 if condition A is satisfied and 0 otherwise. Without loss of generality we posit

[11] In the Hotelling framework, quadratic transportation costs lead to firms that locate apart from each other when locations are chosen prior fares, and to linear and continuous demands and concave profits in both prices at any firm location, see D'Aspremont et al. (1979). This is not the case when consumers' transportation cost is linear. See Anderson (1988) for a thorough treatment of 'linear-quadratic' transport costs. Regarding the impact of non-uniform consumer densities on the agglomeration or deglomeration forces driving firms' location, see Anderson et al. (1997).

that departing early is less costly than departing late for travellers, i.e., $\widehat{\beta} < \widehat{\gamma}$.[12] A travel service (toward the same destination) is not necessarily scheduled at the same time of day across facilities. We treat in the text the case $T_0 \leq T_1$.[13] This departure time configuration allows to classify the travellers according to their departure time preferences into three categories: those with $t \leq T_0$ who prefer to depart earlier than the earliest service offered in the city, those with $t \in]T_0, T_1[$ who may incur early or late schedule delay depending on the chosen facility, and those with $t \geq T_1$ who prefer to depart later than the latest service offered.

Figure 1: Traveller's schedule delay costs

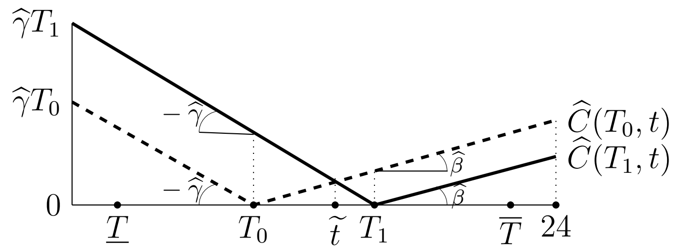


Figure 1 shows the schedule delay costs faced by a consumer for carrier 0 departing at time T_0 and carrier 1 at time T_1. A traveller with desired time $t = 0$ (resp. $t = 24$) incurs the largest schedule delay cost at the facility offering the latest (resp. earliest) departure, i.e., facility 1 (resp. facility 0). The schedule delay cost is null at a facility when consumers desired time matches the time scheduled at that facility ($T_i = t$), and equal across facilities when consumers desired time equals the cost-weighted average $\widetilde{t} = (\widehat{\beta}T_0 + \widehat{\gamma}T_1)/(\widehat{\beta} + \widehat{\gamma})$.

If the service is consumed, the net benefit of travelling for a consumer with desired time t, located at $x \in [0, 1]$, and departing facility i is given by:

$$\widehat{U}_i = \widehat{U} - \widehat{p}_i - \widehat{C}(T_i, t) - \frac{\theta}{2} d_i^2, \quad i = 0, 1,$$

where $\widehat{U}$ represents the gross benefit of the trip in monetary units and $\widehat{p}_i + \widehat{C}(T_i, t)$ is the 'service cost' incurred by the consumer for buying from carrier i, net of her transportation cost to reach the departure facility. We assume that consumer preferences for departure times are independent of the location in the city.[14] The indifferent

[12]This assumption guarantees the existence of a price equilibrium in the model of bottleneck congestion for auto commuting, see Arnott et al. (1993). Small (1982) finds that this assumption is empirically valid for work trips. The evidence for air travel is more mixed, see Lijesen (2006); Warburg et al. (2006); Brey and Walker (2011) or Koster et al. (2014).

[13]The case $T_0 \geq T_1$ is equally valid and developed in Appendix C. We will appeal to it when necessary. The schedule differences arising from the model are discussed in detail in the time game.

[14]This is consistent with Brey and Walker (2011) who find that party size and time zone change are the most influential variables of departure time preferences in air travel.

consumer $\widetilde{x}(t)$ is determined by equalizing $\widehat{U}_0$ with $\widehat{U}_1$, that is:

$$\widetilde{x}(t) = \frac{\widehat{p}_1 + \widehat{C}(T_1, t)}{\theta(1-h)} - \frac{\widehat{p}_0 + \widehat{C}(T_0, t)}{\theta(1-h)} + \frac{1+h}{2}. \tag{1}$$

The number of consumers with desired time t going to facility 0 (rather than 1) decreases in the service cost of its carrier ($\widehat{p}_0 + \widehat{C}_0$), increases in the service cost of the carrier operating at the rival facility ($\widehat{p}_1 + \widehat{C}_1$) and increases with h if the inter-facility transportation cost is larger than the difference in service costs between facility 0 and facility 1.[15] Furthermore, a higher transportation cost parameter (θ) induces more consumers at facility 0 and less of them at the other facility if the service cost at facility 1 is larger, i.e., $\widehat{p}_1 + \widehat{C}_1 > \widehat{p}_0 + \widehat{C}_0$.

Figure 2: Indifferent consumer along the geographic line

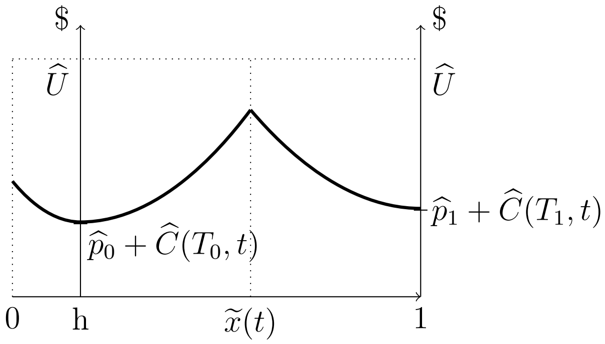


Figure 2 shows the indifferent consumer along with its full fare on the geographic line for facility 0 located at $x = h$ and facility 1 located at $x = 1$. In order for everyone to consume, the gross benefit of the trip of each consumer must offset the full fare at one of the facilities. We say that the market is covered for any given location of the facilities and for any given departure time of their carrier if everyone consumes and if a strictly positive fraction of consumers depart each facility whatever their desired departure time. To ensure that the market is covered, it suffices that the consumers located at $x = 0$ (resp. $x = 1$) with desired departure time $t = T_1$ (resp. $t = T_0$) chooses facility 0 (resp. facility 1), or

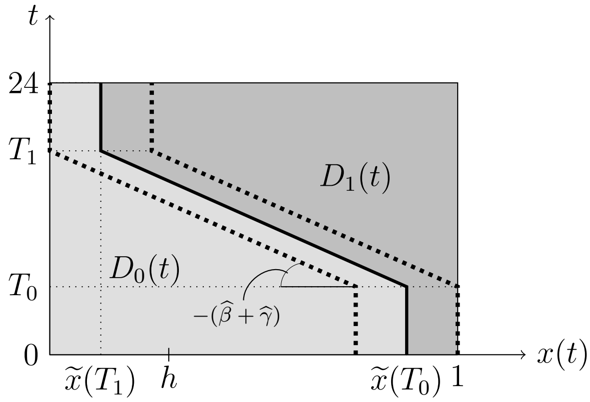
$$-\frac{\theta}{2}(1-h^2) < \widehat{p}_1 - \widehat{p}_0 - \widehat{\beta}(T_1 - T_0) \leq \widehat{p}_1 - \widehat{p}_0 + \widehat{\gamma}(T_1 - T_0) < \frac{\theta}{2}(1-h)^2. \tag{2}$$

The first inequality states that the service cost at facility 0 shall not be too large to induce those situated at $x = 0$ with desired time T_1 or later to depart facility 0.

[15] To show this, set $\partial\widetilde{x}(t)/\partial h > 0$ and rearrange to get $\frac{\theta}{2}(1-h)^2 > (\widehat{p}_0 + \widehat{C}_0) - (\widehat{p}_1 + \widehat{C}_1)$. Note that setting $\widetilde{x}(t) > h$ yields the same result. In contrast to the Hotelling model with linear transportation costs, our setup allows potential consumers located at the left-hand side of facility 0 on the geographic line to consume the homogeneous product at facility 1 if its service cost is low enough.

The last inequality requires the inter-facility transportation cost to be large enough to lead those located at $x = 1$ with desired time T_0 or earlier to depart facility 1. Both conditions are maintained hereafter.[16] Fig. 3 illustrates the covered market condition and establishes the link between the geographic and time dimensions.

Figure 3: Indifferent consumer and the time line



For given fares, departure times, parameters and given a desired time t, the travellers located to the left-hand side (LHS) of the solid black *indifference line* on Fig. 3 depart facility 0 and those to the right-hand side (RHS) depart facility 1. By way of example, when the indifferent consumer prefers departing at time T_0 or earlier, $\widetilde{x}(t) = \widetilde{x}(T_0)$ and a larger share of consumers choose facility 0 located at $x = h$ and a positive fraction of them, $1 - \widetilde{x}(T_0)$, leave from facility 1 located at $x = 1$. Integrating over the geographic space for any given t, we get the demand functions $D_0(t) = \widetilde{x}(t)$ and $D_1(t) = 1 - \widetilde{x}(t)$. The broken shape of the indifference line is related to the piecewise schedule delay cost function $\widehat{C}_i(T_i, t)$. When $\widehat{p}_0$ (resp. $\widehat{p}_1$) increases, all else equal, the indifference line moves to the LHS (resp. RHS). From Fig. 3, we deduce the following lemma.

Lemma 1. *If the consumers located at $x = 0$ (resp. $x = 1$) with desired departure time $t = T_0$ (resp. $t = T_1$) select facility 1 (resp. facility 0), then no consumer chooses facility 0 (resp. facility 1).*

Given a density $\rho(t)$ of desired departure times, aggregating the individual demands over the geographic and time lines, we obtain the following the market demand at each

[16]See Appendix B.2 for the derivation of the covered market condition (2). This condition could be relaxed to allow all travellers with desired departure time $t \geq T_1$ (resp. $t \leq T_0$) to depart facility 1 (resp. facility 0). These situations are not analyzed here but Appendix B.7 discusses the parameter conditions under which these cases are more likely to happen.

facility:[17]

$$
\begin{aligned}
D_0(\mathbf{p}, \mathbf{T}) &= \int_0^{24} \widetilde{x}(t)\rho(t)\,\mathrm{d}t = p_1 - p_0 + \frac{1+h}{2} + \Phi(\mathbf{T}), \\
D_1(\mathbf{p}, \mathbf{T}) &= 1 - D_0(\mathbf{p}, \mathbf{T}),
\end{aligned}
\tag{3}
$$

where $\mathbf{p} \equiv (p_0, p_1)$ with $p_0 = \widehat{p}_0/\theta(1-h)$, $p_1 = \widehat{p}_1/\theta(1-h)$ and $\mathbf{T} \equiv (T_0, T_1)$. In what follows, we often divide the fares and schedule delay cost parameters by $\theta(1-h)$ and 'drop the hats' to get more compact expressions.[18] Term $\Phi(\mathbf{T})$, defined as

$$
\Phi(\mathbf{T}) = \gamma(T_1 - T_0)m_\ell + (\beta T_0 + \gamma T_1)m_c - \beta(T_1 - T_0)m_r - (\beta + \gamma)\bar{t}_c \tag{4}
$$

with $\beta = \widehat{\beta}/\theta(1-h)$ and $\gamma = \widehat{\gamma}/\theta(1-h)$, captures the (normalized) difference in schedule delay costs (shorthanded SDC hereafter) at the market level and aggregates the individual SDC differences between facility 1 and facility 0 through the shares $m_\ell = \int_0^{T_0} \rho(t)dt$, $m_c = \int_{T_0}^{T_1} \rho(t)dt$ and $m_r = \int_{T_1}^{24} \rho(t)dt$, and through an average desired time $\bar{t}_c = \int_{T_0}^{T_1} t\rho(t)dt$. Clearly, a positive (resp. negative) term $\Phi(\mathbf{T})$ gives facility 0 (resp. facility 1) and its carrier a SDC advantage that renders facility 0 (resp. facility 1) more attractive to travellers. When $T_0 = T_1 = T$, the SDC advantage is null, i.e., $\Phi(T, T) = 0$. As the SDC term will play a central role in the analysis, the following lemma mentions some of its properties.

Lemma 2. *Consider a travel service scheduled at time T_0 (resp. T_1) at facility 0 (resp. facility 1) with $T_0 \leq T_1$. Let F(t) be the CDF of consumers desired departure time with $t \in [0, 24]$. Then:*

$$
\begin{aligned}
\Phi_{T_0} &= \beta - (\beta + \gamma)m_\ell, & \Phi_{T_1} &= \gamma - (\beta + \gamma)m_r, \\
\Phi_{T_0,T_0} &= -(\beta + \gamma)\rho(T_0) < 0, & \Phi_{T_1,T_1} &= (\beta + \gamma)\rho(T_1) > 0,
\end{aligned}
$$

where $\Phi_{T_i} \equiv \partial\Phi(\mathbf{T})/\partial T_i$ and $\Phi_{T_i,T_i} \equiv \partial^2\Phi(\mathbf{T})/\partial {T_i}^2$. Setting Φ_{T_i} larger than zero yields

$$
\Phi_{T_0} > 0 \quad \textit{iff} \quad T_0 < F^{-1}\left(\frac{\beta}{\beta+\gamma}\right), \quad \Phi_{T_1} > 0 \quad \textit{iff} \quad T_1 > F^{-1}\left(\frac{\beta}{\beta+\gamma}\right).
$$

Proof. See Appendix B.3.

Lemma 2 establishes that function $\Phi(\mathbf{T})$ is concave in T_0 and convex in T_1 and Appendix C.3 further shows that it remains so when $T_0 \geq T_1$. Then, the SDC advantage of facility 0 increases/decreases in T_0 when the service scheduled by its carrier is below/above the $\beta/(\beta+\gamma)$-th quantile of consumers desired departure time distribution.

[17] As shown in Fig. 3, the integrand $\tilde{x}(t)$ in (3) is piecewise linear on the time segments $[0, T_0]$, $]T_0, T_1[$ and $[T_1, 24]$ which implies integrating a different expression on each subdomain. See Appendix B.1 for a detailed derivation of the market demands.

[18] The scaling by $1/\theta(1-h)$ is innocuous in all derivations.

When facility 0's SCD advantage expands, more travellers choose facility 0 and less of them depart facility 1. Similarly, the SDC disadvantage of facility 1 increases/decreases in T_1 when T_1 is above/below time $F^{-1}[\beta/(\beta+\gamma)]$ and less/more travellers select facility 1. Given that $\beta < \gamma$, time $F^{-1}[\beta/(\beta+\gamma)]$ lies below consumers' median desired departure time and it will be closer to the boundaries or to the centre of the time line depending on the shape of $F(t)$. As the uniform distribution is often used as a tractable benchmark in Hotelling's settings, note that when $\rho(t) = \mathcal{U}[0, 24]$ then $\Phi(\mathbf{T})$ becomes

$$\Phi^U(\mathbf{T}) = \frac{\beta+\gamma}{48}T_1^2 + \beta(T_0 - T_1) - \frac{\beta+\gamma}{48}T_0^2 \tag{5}$$

and $F^{-1}[\beta/(\beta+\gamma)] = 24\beta/(\beta+\gamma)$. Furthermore, $\Phi^U(\mathbf{T})$ is identical whether $T_0 \leq T_1$ or $T_0 \geq T_1$.[19]

We now propose some comparative statics to analyze the effects of a change in fares, departure times and in the location advantage of facility 0 on consumer demands. We deduce from (3) that demand at a facility decreases in the fare of its carrier and increases in the fare of the carrier serving the rival facility. Differentiating consumer demands with respect to T_i and h, we obtain:

$$\begin{aligned} &\frac{\partial D_0(\mathbf{p},\mathbf{T})}{\partial T_i} = \Phi_{T_i}(\mathbf{T}), \quad \frac{\partial D_1(\mathbf{p},\mathbf{T})}{\partial T_i} = -\frac{\partial D_0(\mathbf{p},\mathbf{T})}{\partial T_i} \quad \text{for } i = 0, 1, \\ &\frac{\partial D_0(\mathbf{p},\mathbf{T})}{\partial h} = \frac{1}{2} + \frac{p_1 - p_0 + \Phi(\mathbf{T})}{(1-h)}, \quad \frac{\partial D_1(\mathbf{p},\mathbf{T})}{\partial h} = -\frac{\partial D_0(\mathbf{p},\mathbf{T})}{\partial h}. \end{aligned} \tag{6}$$

Demands' sensitivity to changes in departure times follows directly from the discussion under Lemma 2. Similarly to the results related to h under (1), setting $\partial D_i/\partial h > 0$ for $i = 0, 1$ and rearranging, we can show that consumers' demand at facility 0/facility 1 increases with h if the inter-facility transportation cost is larger/lower than the difference in average service cost between facility 0 and facility 1.[20] Equalizing the departure times and fares across carriers, $D_0 = (1+h)/2$ while $D_1 = (1-h)/2$. Having characterized consumers' problem, we turn to model the competition between carriers.

2.2 Carrier-rivalry game

Two carriers compete with each other across facilities to attract consumers. This section focuses on the time-then-fare subgame of the three-stage game. Setting departure times before fares is the most common behaviour for carriers offering (air, rail,

[19] That is, (4) is identical to (C.3) in the uniform case. Said differently, by using Appendix C's notation, we have $\Phi^U(\mathbf{T}) \equiv \Phi^{sym,U}(\mathbf{T})$.

[20] Moreover, a higher transportation cost parameter induces a larger demand at facility 0 and lower one at the other facility if the service cost of facility 1 is larger than that of facility 0. See Eqs. (B.7) in the Appendix for further details.

road, water) transportation services to individuals, in particular for long-distance trips. Moreover, fares are generally easier to adjust than departure times. The two-stage decision process is solved backward: we first maximize carrier profits with respect to fares as functions of the departure times, and then analyze carrier timing decisions.

2.2.1 The fare game

Service timing in passenger transportation is often regulated by transport authorities to improve the social welfare or to correct market failures (limited capacity of the facilities, nuisance to neighbours, labour regulation, safety). This section analyzes carriers' market when departure times are given to carriers. We assume that carriers' operational costs and facility fees are directly expressed per passenger. Furthermore, we consider that carriers' operational time costs along the time of day are separable from other costs and we set carriers' fixed cost to zero. Then, the profit function of each carrier is:

$$\widehat{\pi}_i(\mathbf{p}, \mathbf{T}, \boldsymbol{\tau}) = (\widehat{p}_i - \widehat{c}_i - \widehat{\tau}_i) D_i(\mathbf{p}, \mathbf{T}) - \widehat{K}(T_i), \quad i = 0, 1, \tag{7}$$

where $\widehat{c}_i \geq 0$ is the marginal operational cost of the carrier located at facility i, $\widehat{\tau}_i \geq 0$ is the per passenger fee charged by facility i to its carrier and $\widehat{K}(T_i) \geq 0$ is the operational total time cost incurred by carrier i for departing its facility at a feasible time T_i along the day.[21] Given the departure times $\mathbf{T}$ and the fare of their rival, each carrier sets its fare to maximize its profit. Solving the system of first-order conditions (FOCs) $\partial \widehat{\pi}_i / \partial \widehat{p}_i = 0$ for $i = 0, 1$ with respect to the fares leads to:

$$\begin{aligned} p_0^* &= \frac{2}{3}(c_0 + \tau_0) + \frac{1}{3}(c_1 + \tau_1) + \frac{3+h}{6} + \frac{1}{3}\Phi(\mathbf{T}), \\ p_1^* &= \frac{2}{3}(c_1 + \tau_1) + \frac{1}{3}(c_0 + \tau_0) + \frac{3-h}{6} - \frac{1}{3}\Phi(\mathbf{T}), \end{aligned} \tag{8}$$

where p_i^*, c_i and τ_i for $i = 0, 1$ stand for $\widehat{p}_i^*, \widehat{c}_i$ and $\widehat{\tau}_i$ divided by $\theta(1-h)$. The resulting vector of (normalized) *equilibrium* fares, denoted $\mathbf{p}^* \equiv (p_0^*, p_1^*)$, represents a Nash equilibrium.[22] The first two terms on the RHS of Eqs. (8) are the (normalized) marginal costs of each carrier plus the usual duopolistic markup/markdown which is proportional to the marginal costs of the rival carrier serving the other facility. The third terms are a monopoly premium/penalty stemming from the location advantage/disadvantage of the facility at which a carrier operates. The last terms represent a markdown/markup related to the (normalized) SDC (dis)advantage of the carrier.

[21] Carriers' time costs don't play much of a role at the pricing stage but further details are provided in the time game.

[22] The existence of the Nash equilibrium follows from the concavity in fares of the profit functions. Its uniqueness and stability are easy to verify here (see Vives, 1999, pp. 47-52). Fares can also be shown to be strategic complements in carrier decisions.

Fare expressions (8) can be further rearranged to analyze the difference in pricing and markup across carriers. Focusing first on the difference in (normalized) fares, setting p_0^* strictly larger than p_1^* and rearranging yields: $p_0^* > p_1^*$ *iff*

$$\Delta\widetilde{c} < h + 2\Phi(\mathbf{T}), \tag{9}$$

where $\Delta\widetilde{c} = (c_1+\tau_1)-(c_0+\tau_0)$ represents the (normalized) marginal costs (dis)advantage of carrier 0 with respect to its rival at the other facility when $\Delta\widetilde{c} > 0$ ($\Delta\widetilde{c} < 0$). Equalizing the departure times across facilities and assuming no location advantage[23] for facility 0 in (9), carrier 0 charges a higher fare than its rival if its marginal costs are higher ($\Delta\widetilde{c} < 0$). In commercial aviation, this is typically the case of a legacy carrier that competes over a shared market with a lower marginal costs carrier departing the other facility. Setting $h > 0$, the location advantage of facility 0 allows its carrier to charge a higher fare than its rival, even if it has *lower* marginal costs ($\Delta\widetilde{c} > 0$). Hence, in equilibrium, a carrier benefiting from the better location of its facility and with lower marginal costs than a rival carrier serving the other facility will charge a higher fare than its rival, if its advantage in location is large enough. The SDC term has a similar impact on carriers' fare as the location advantage and can strengthen or reduce a geographic (dis)advantage. The following proposition summarizes the above results.

Proposition 1. *Consider carrier* 0 *(resp.* 1*) that compete in fares on a linear geographic market with fixed scheduled times* T_0 *(resp.* T_1*), with* $T_0 \leq T_1$*. There exists a unique Nash equilibrium in fares given by* (8)*. In equilibrium, carrier* 0 *charges a higher fare than its rival if* (9) *holds, that is, if its location and schedule delay cost advantages offset its marginal costs advantage.*

To show that Proposition 1 holds when $T_0 \geq T_1$, use demands (C.2) in the profit functions (7) and apply the same derivation steps. The SDC difference $\Phi(\mathbf{T})$ in the related expressions will be replaced by its counterpart $\Phi^{sym}(\mathbf{T})$ given in Eq. (C.3).

Defining the (normalized) markup of carrier i in equilibrium as $m_i^* = p_i^* - (c_i + \tau_i)$ for $i = 0, 1$, setting m_0^* strictly larger than m_1^* and rearranging yields: $m_0^* > m_1^*$ *iff*

$$\Delta\widetilde{c} > -\frac{h}{2} - \Phi(\mathbf{T}). \tag{10}$$

Assuming no location and SDC advantages, the first terms in (10) becomes positive which means that the markup of carrier 0 exceeds its rival's markup if its marginal costs are lower. Further setting $h > 0$, the advantage in location of facility 0 allows its carrier to receive a higher markup than its rival, even if its carrier has *higher* marginal costs ($\Delta\widetilde{c} < 0$) than its competitor at the other facility. Therefore, the markup of

[23] In the whole paper, the expressions 'no location advantage' or 'dropping the location advantage' mean setting $h = 0$. Note that $h = 0$ implies *maximum* geographic differentiation between the facilities and their carrier while $T_0 = T_1$ implies *minimum* schedule differentiation.

a carrier serving a facility endowed with a better location can be higher than the markup of the rival lower marginal costs carrier if its marginal costs disadvantage is not excessive. Considering the full expression (10), the SDC advantage has a similar impact on carrier markups as the location advantage and can either expand or reduce the location advantage at facility 0.

We can explore the impact of an increase in T_i and in h on the equilibrium fares with the following expressions:

$$\frac{\partial p_0^*}{\partial T_i} = \frac{1}{3}\Phi_{T_i}(\mathbf{T}), \quad \frac{\partial p_1^*}{\partial T_i} = -\frac{\partial p_0^*}{\partial T_i}, \quad i = 0, 1, \tag{11}$$

$$\frac{\partial \widehat{p}_0^*}{\partial h} = -\frac{\theta(1+h)}{3} < 0, \quad \frac{\partial \widehat{p}_1^*}{\partial h} = -\frac{\theta(2-h)}{3} < 0. \tag{12}$$

The sensitivity of the (normalized) fares to changes in departure times can be analyzed in light of Lemma 2: p_0^* increases in T_0 (resp. T_1) if the service of the carrier serving facility 0 (resp. facility 1) is scheduled earlier (resp. later) than the $\beta/(\beta+\gamma)$th quantile of consumers' desired time distribution. The reasoning for p_1^* is analogous. Next, notice that (12) are calculated on $\widehat{p}_i^*$ and not on p_i^*.[24] As $h \in [0, 1[$, the equilibrium fares are decreasing in h at both facilities. A shorter inter-facility distance enhance the rivalry between the carriers and reduces the equilibrium fares at both facilities.

Substituting the equilibrium fares $\mathbf{p}^*$ in consumer demands (3), carriers' marginal costs become explicit in the *equilibrium* demands, that is:

$$D_0^*(\mathbf{T}, \boldsymbol{\tau}) = \frac{1}{6}[3 + h + 2\Delta\widetilde{c} + 2\Phi(\mathbf{T})], \quad D_1^*(\mathbf{T}, \boldsymbol{\tau}) = 1 - D_0^*(\mathbf{T}, \boldsymbol{\tau}), \tag{13}$$

where $D_0^* \in]0, 1[$ requires $|\frac{1}{3}[h + 2\Delta\widetilde{c} + 2\Phi(\mathbf{T})]| < 1$. As outlined in Basso and Zhang (2007), in equilibrium, the marginal costs of a carrier have the same effects on the equilibrium demand as those of fares: higher marginal costs for a carrier induce a lower equilibrium demand at its facility and a larger one at the rival facility. Dropping the location and SDC advantage terms, and equalizing marginal costs across carriers, the market demand is evenly shared across facilities, as expected. A larger demand at a facility goes hand-in-hand with a larger markup for its carrier than its rival's markup at the other facility.[25]

Regarding the effects of a change in T_0 and T_1 on the equilibrium demands, one can readily see in (13) that $\partial D_0^*(\mathbf{T}, \boldsymbol{\tau})/\partial T_i$ and $\partial D_1^*(\mathbf{T}, \boldsymbol{\tau})/\partial T_i$ for $i = 0, 1$ leads to, respectively, $\partial p_0^*/\partial T_i$ and $\partial p_1^*/\partial T_i$ in (11). Thus, departure times have exactly the same impact on the equilibrium demands as on the equilibrium fares. Next, differentiating the equilibrium demands with respect to h, setting the resulting expression larger than

[24]Recall that $\widehat{p}_i^* = p_i^*\theta(1-h)$ and that c_i, τ_i and $\Phi(\mathbf{T})$ in (8) are divided by $\theta(1-h)$. With this in mind, derivatives (12) are straightforward.

[25] To prove this, use (13), set $D_0^* > D_1^*$ and rearrange to get (10).

zero and rearranging, we obtain:[26]

$$\frac{\partial D_0^*(\mathbf{T},\boldsymbol{\tau})}{\partial h} > 0 \quad iff \quad \frac{\theta}{2}(1-h)^2 > -\Delta\widehat{\widetilde{c}} - \widehat{\Phi}(\mathbf{T}), \tag{14}$$

where $\Delta\widehat{\widetilde{c}} = \theta(1-h)\Delta\widetilde{c}$ and $\widehat{\Phi}(\mathbf{T}) = \theta(1-h)\Phi(\mathbf{T})$, and the reverse holds for $\partial D_1^*/\partial h$. Thus, if the carrier that benefits from a location advantage (carrier 0) is more competitive in marginal and schedule delay costs than its rival (both $\Delta\widehat{\widetilde{c}}$ and $\widehat{\Phi}(\mathbf{T}) > 0$), increasing h increases the demand at its facility at the expenses of its rival at the other facility. If carrier 0 is less competitive in marginal and schedule delay costs (both $\Delta\widehat{\widetilde{c}}$ and $\widehat{\Phi}(\mathbf{T}) < 0$), increasing h increases demand at its facility if the inter-facility transportation cost is larger than carrier 0's marginal and schedule delay costs disadvantages. Taking the perspective of the carrier serving the most remote facility, carrier 1 captures part of its 'rival's backyard' when h increases if its marginal costs and SDC advantages fully compensate the transportation cost from facility 0 to facility 1.

In equilibrium, the profits (7) can be written as:

$$\pi_i^*(\mathbf{T},\boldsymbol{\tau}) = D_i^{*2}(\mathbf{T},\boldsymbol{\tau}) - K(T_i), \quad i = 0, 1, \tag{15}$$

where $\pi_i^* = \widehat{\pi}_i/\theta(1-h)$ and $K(T_i) = \widehat{K}(T_i)/\theta(1-h)$. As expected, the *equilibrium* profit of a carrier depends upon all determinants of the equilibrium demand at its facility (in particular the equilibrium fare, the fee and the departure time set at the other facility) minus carrier's own total operational time costs. Ignoring the latter term for now and setting $D_0^{*2} > D_1^{*2}$, we can readily exploit a result obtained for the equilibrium demands under (13): $\pi_0^* > \pi_1^*$ if the markup of carrier 0 exceeds carrier 1's markup. Thus, a larger markup for a carrier implies a larger demand and profit than its rival at the other facility when carriers' time costs are null. The following proposition summarizes these results.

Proposition 2. *Consider carrier* 0 *(resp.* 1*) departing their facility at times* T_0 *(resp.* T_1*), with* $T_0 \leq T_1$ *and let* $K(T_0) = K(T_1) = 0$*. In equilibrium, the markup fare of carrier* 0*, its demand and profit are higher than its rival's if* (10) *holds, that is, if its marginal costs disadvantage does not offset its location and schedule delay costs advantages.*

Again, to prove that Proposition 2 holds when $T_0 \geq T_1$, use the symmetric demands (C.2) in the profit functions (7) and follow the same derivation steps. The SDC difference $\Phi(\mathbf{T})$ is replaced by its counterpart $\Phi^{sym}(\mathbf{T})$ given in Eq. (C.3).

Turning to the comparative statics of the profits with respect to T_i and h, we get:

$$\frac{\partial \pi_i^*(\mathbf{T},\boldsymbol{\tau})}{\partial T_j} = \frac{2}{3}\Phi_{T_j} D_i^*(\mathbf{T},\boldsymbol{\tau}) - \frac{\partial K(T_i)}{\partial T_i}, \quad \text{for } i,j = 0,1 \text{ and for } i \neq j, \tag{16}$$

[26] Replacing the difference in carriers' marginal costs by the difference in fares, (14) is identical to $\partial D_0(\mathbf{p},\mathbf{T})/\partial h$ in Eqs. (B.7) of the Appendix.

$$
\begin{aligned}
\frac{\partial \widehat{\pi}_0^*(\mathbf{T}, \boldsymbol{\tau})}{\partial h} &= \frac{\theta}{3}\left[-\frac{(1+3h)}{2} + \Delta\widetilde{c} + \Phi(\mathbf{T})\right] D_0^*(\mathbf{T}, \boldsymbol{\tau}), \\
\frac{\partial \widehat{\pi}_1^*(\mathbf{T}, \boldsymbol{\tau})}{\partial h} &= \frac{\theta}{3}\left[\frac{3h-5}{2} - \Delta\widetilde{c} - \Phi(\mathbf{T})\right] D_1^*(\mathbf{T}, \boldsymbol{\tau}).
\end{aligned}
\tag{17}
$$

Regarding the impact of a change in the departure times on carrier profits in (16), ignoring $\partial K(T_i)/\partial T_i$, we notice that $\text{sgn}\left(\partial \pi_i^*/\partial T_j\right) = \text{sgn}\left(\Phi_{T_j}\right)$. Hence, Lemma 2 applies again. The effect of the location of facility 0 on carrier profits is explored through the "competition" and "demand" effects.[27] Whether an increase in demand at *one* facility compensates the decrease in the fare of its carrier when h increases depends on the relative (dis)advantages identified above. Fig. 4 summarizes the net effect of a RHS move of h on carrier profits along the $\Delta\widetilde{c}$ line.

Figure 4: Effect on π_i^* for a right move of h.

$$
\begin{array}{ccccc}
\downarrow \widehat{\pi}_0^*, \uparrow \widehat{\pi}_1^* & & \downarrow \widehat{\pi}_0^*, \downarrow \widehat{\pi}_1^* & & \uparrow \widehat{\pi}_0^*, \downarrow \widehat{\pi}_1^* \\
\longrightarrow & | & \longrightarrow & | & \longrightarrow \; \Delta\widetilde{c} \\
& \frac{3h-5}{2} - \Phi(\mathbf{T}) & & \frac{1+3h}{2} - \Phi(\mathbf{T}) &
\end{array}
$$

Assuming strictly positive transportation cost parameter θ and equilibrium demands, we can focus on the terms within brackets in (17) to determine under which conditions the demand effect dominates the competition effect when h expands. We deduce that $\widehat{\pi}_0^*$ increases in h if $\Delta\widetilde{c} > (1+3h)/2 - \Phi(\mathbf{T})$. Hence, carrier 0 needs a large marginal costs advantage ($\Delta\widetilde{c} > (1+3h)/2$) to increase its profit when h increases, unless its SDC advantage is large enough ($\Phi(\mathbf{T}) > (1+3h)/2$). The same interpretation holds regarding the condition needed for the profit of carrier 1 to increase in h, i.e., $\Delta\widetilde{c} < (3h-5)/2 - \Phi(\mathbf{T})$.

Having characterized carriers' problem with exogenous departure times, we proceed to consider the time game for a simultaneous choice of the departure times.

2.2.2 The time game

At this stage, departure times become strategic variables for the carriers and accounting for their operational time costs is fundamental to understand their schedule decisions.

Scheduling a transport service at the most appropriate time of the day is a central element of carriers' planning. In commercial aviation, a large number of operational research professionals have been developing methods to optimize airline schedules since the 1950s. Etschmaier and Mathaisel (1985) describe the simplest model solved in mathematical programming as follows: given (i) a set of demand functions and associated revenues for every passenger origin-destination pair market over the time of day

[27]The competition and demand effects related to h are given by $\frac{\partial \widehat{\pi}_i^*}{\partial h} = D_i^* \frac{\partial \widehat{p}_i^*}{\partial h} + (\widehat{p}_i^* - \widehat{c}_i)\frac{\partial D_i^*}{\partial h}$.

(and the day-of-week of the planning cycle), (*ii*) route characteristics (distance, times and operating restrictions), (*iii*) aircraft characteristics and operating costs, and (*iv*) operating and managerial constraints; find a set of flights with associated assignments of aircraft *and times of departure and arrival* which maximize profits. In every day's operations, optimized schedules are rarely executed as planned due to unexpected disruptions of the transport service (weather conditions, unscheduled maintenance, unavailable crews, congested airports) that propagate through the network. Accounting explicitly for these uncertainties and network effects goes beyond the scope of our model so we focus on the most relevant features that our setup can capture. Clearly, carriers are not always free to locate their departure or arrival times where they wish on the 24-hour clock and departure/arrival times are often constrained to lie during facilities operating hours (night operating restrictions). Scheduling a transport service at a specific time of the day may be more or less costly in terms of logistics, and may depend on carriers' business model.[28] The timing of the service can also be subject to important direct costs, such as peak/off-peak charges or slot acquisition costs in congested or coordinated airports, which add up to other operational costs that do not depend directly on the time of departure (fuel, overflight charges).[29]

We assume that carriers can decompose their operational time costs $K(T_i)$ additively into a fixed component and a cost that varies along the time of day over all feasible service times $T_i \in [\underline{T}, \overline{T}]$ at their departure facility. We posit that the time-varying component is either null or linearly increasing (or decreasing) in T_i. This simple functional form allows to explore analytically the effect of the marginal time costs on carriers' timing decisions.

[28] In air transportation, low-cost airlines typically serve short haul (<3h), point-to-point routes from/to secondary (and less congested) airports while legacy carriers generally operate medium (3h-6h) or long haul (>6h) flights in hub-and-spoke networks connected to primary (and more congested) airports. The business of low-cost airlines is often characterized by a more intense daily use of their fleet as compared to legacy carriers, shorter turnaround time between operations, and aircraft and crew returned to a base airport, which reduces aircraft maintenance costs or overnight accommodation costs, see IATA (2005) or Gross and Schroeder (2007). This, in turn, favours the scheduling of flights at the earliest and latest available times at the airports they serve; see Bley and Buermann (2007, pp.59-62) for a discussion on the strategic use of schedules by low-cost airlines to reduce their operational costs.

[29] Peak/off-peak landing or taking off fees are usually charged per weight and/or aircraft type. Greater Toronto Airports Authority (2016) charges to fixed wing aircrafts of 19000 kg (or less) a fee of $145 per 1000 kg of maximum permissible takeoff weight during the peak period (Mon-Fri 0700-1000 & Sun-Fri 1430-2100) while the non-peak period fee is $82.50 per 1000 kg. The process for acquiring a slot under IATA regulation is described in Ulrich (2008) and Gillen (2008), and involves a well regulated but complex bargaining process between parties (slot coordinator, carriers, airport authorities) that results in slot swaps between carriers, leases, new slot allocation or slot trading depending on the regulatory frameworks. These costs are far from being negligible at congested or coordinated airports. As reported by Done (2008), Continental Airlines paid $116m for its summer slots at Heathrow and planned further $93m in winter.

Figure 5: Carriers' operational time costs as a function of the time of day

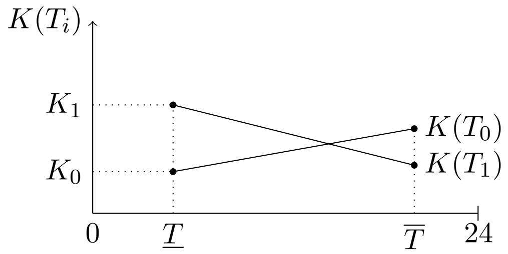


Fig. 5 shows the case of a linear and increasing time cost along the time of day for the carrier serving facility 0, and a linearly decreasing one for the carrier operating at facility 1. Clearly, a positive marginal time cost for a carrier favours a departure time closer to the opening hour of its facility while a negative one favours a service scheduled closer to the closing hour. Heterogeneity in consumers' desired time may prevent such extreme outcomes. Other configurations of marginal time costs (such as positive or negative for both carriers) follow the same reasoning. Let carriers' total operational time costs be given by:

$$\begin{aligned} K(T_0) &= K_0 + k_0\ T_0, \quad \text{with} \quad K_0 \geq 0, \quad k_0 \neq 0, \quad T_0 \in [\underline{T}, \overline{T}], \\ K(T_1) &= K_1 + k_1\ T_1, \quad \text{with} \quad K_1 \geq 0, \quad k_1 \neq 0, \quad T_1 \in [\underline{T}, \overline{T}], \end{aligned} \tag{18}$$

where $K(T_i) = \widehat{K}(T_i)/\theta(1-h)$ and assume that the facilities set their opening and closing hours to allow their carrier to depart at their optimal time along the day, i.e., $\underline{T}$ and $\overline{T}$ are not binding. Given the endogenous fares (8), we can focus on maximizing profits (15) with respect to T_i. Combining (16) with (18) and assuming for now the existence of interior and unique solutions on $[0, 24]$, setting $\partial\pi_i^*/\partial T_i = 0$ for $i = 0, 1$ yields the following first-order conditions (FOCs):

$$\begin{aligned} \frac{\partial \pi_0^*(\mathbf{T}, \boldsymbol{\tau})}{\partial T_0} &= \frac{2}{3}\left[\beta - (\beta+\gamma)m_\ell\right] D_0^*(\mathbf{T}, \boldsymbol{\tau}) - k_0 = 0, \\ \frac{\partial \pi_1^*(\mathbf{T}, \boldsymbol{\tau})}{\partial T_1} &= -\frac{2}{3}\left[\gamma - (\beta+\gamma)m_r\right] D_1^*(\mathbf{T}, \boldsymbol{\tau}) - k_1 = 0, \end{aligned} \tag{19}$$

and, by rearranging these expressions, we can characterize the candidate departure times by the following implicit functions:

$$F(T_i) = \frac{\beta}{\beta+\gamma} - \frac{3k_i}{2(\beta+\gamma)D_i^*(\mathbf{T}, \boldsymbol{\tau})}, \quad i = 0, 1, \tag{20}$$

where $F(T_0) \equiv m_\ell$, $F(T_1) \equiv 1 - m_r$ and $k_i = \partial K(T_i)/\partial T_i$ denotes carrier i's (normalized) marginal time cost along the time of day. The second-order conditions (SOCs) that need to be satisfied are:

$$\begin{aligned}\frac{\partial \pi_0^{*2}(\mathbf{T},\boldsymbol{\tau})}{\partial T_0^2} &= -\frac{2}{3}(\beta+\gamma)\rho(T_0)D_0^*(\mathbf{T},\boldsymbol{\tau}) + \frac{2}{9}[\beta-(\beta+\gamma)\ m_\ell]^2 < 0,\\ \frac{\partial \pi_1^{*2}(\mathbf{T},\boldsymbol{\tau})}{\partial T_1^2} &= -\frac{2}{3}(\beta+\gamma)\rho(T_1)D_1^*(\mathbf{T},\boldsymbol{\tau}) + \frac{2}{9}[\gamma-(\beta+\gamma)\ m_r]^2 < 0.\end{aligned} \tag{21}$$

Consider first the case where carriers' time cost is constant along the time of day ($k_i = 0$ for $i = 0, 1$). FOCs (20) lead to the best timing response functions of each carrier to their rival's schedule, and solving these yields:

$$F(T_0) = F(T_1) = \frac{\beta}{\beta+\gamma} \Rightarrow T^*|_{k_0=k_1=0} = F^{-1}\left(\frac{\beta}{\beta+\gamma}\right). \tag{22}$$

Clearly, the response functions in (22) are identical across carriers and depend only on consumers' unit schedule delay costs and on the CDF $F(t)$. As the equilibrium departure time of a carrier does not depend on the schedule of its rival, each carrier has a dominant strategy which represents a unique and interior Nash equilibrium. Carrier 0 schedules its service to maximize its SDC advantage while carrier 1 set its departure time to minimize its SDC disadvantage. Moreover, the last RHS terms in the SOCs (21) is null when the FOCs are satisfied and the parameter restrictions below (13) become $D_0^*(\mathbf{T}^*,\boldsymbol{\tau}) \in]0,1[$ if $|\frac{1}{3}[h+2\Delta\widetilde{c}]| < 1$. When these restrictions hold and given that $\rho(T_i), \beta$ and γ are positive, this solution maximizes carrier profits.[30]

Turning to the case where carriers' time costs vary with the time of day ($k_i \neq 0$ for $i = 0, 1$), assuming that the solution of system (20) $-$ a system of two nonlinear functions in T_i $-$ yields a unique intersection at coordinate $\mathbf{T}^* = (T_0^*, T_1^*) \in]0, 24[^2$ with $T_i^* \equiv T_i^*(\boldsymbol{\tau}, k_i)$, we need to ensure that this equilibrium maximizes carrier profits. Using the SOCs (21), the last terms on the RHS no longer vanish at the stationary points and profit maximization requires bounded marginal time costs such that:

$$k_i^2 < \frac{4}{3}\rho(T_i^*)(\beta+\gamma)D_i^{*3}(\mathbf{T}^*,\boldsymbol{\tau}), \quad i = 0, 1, \tag{23}$$

where k_i is on both sides of the inequality. In what follows we posit that k_i satisfies (23) and yields a unique and interior Nash equilibrium.[31]

[30]When $k_i = 0$ for $i = 0, 1$, it is straightforward to show that derivatives (19) are positive for all $T_i < T^*|_{k_0=k_1=0}$ and negative for all $T_i > T^*|_{k_0=k_1=0}$, which rules out corner solutions at $T_i = 0$ or $T_i = 24$.

[31]When $k_i \neq 0$ for $i = 0, 1$, the existence, uniqueness and stability of the Nash equilibrium in departure times on $]0, 24[^2$ must be investigated numerically. Assuming that this equilibrium exists and is unique, setting $0 < F(T_i^*) < 1$ for $i = 0, 1$ we can derive bounds around k_i *conditional upon* $D_0^*(\mathbf{T}^*,\boldsymbol{\tau}) \in]0, 1[$ and which will depend on $D_0^*(\mathbf{T}^*,\boldsymbol{\tau})$. These bounds are available upon request. In Appendix B.4, we derive explicit bounds for k_i as well as *existence conditions* for interior solutions in departure times when $\rho(t) = \mathcal{U}[0, 24]$. Section 4.2 further explores numerically the time game in the latter context.

Given that $F(t)$ is a strictly increasing in its argument, we can use (20) to characterize all possible schedule configurations across carriers in equilibrium with respect to $T^*|_{k_0=k_1=0}$. By applying the implicit function theorem to (20), we can show that the equilibrium departure times T_i^* for $i = 0, 1$ are decreasing in k_i when (23) holds.[32] Next, we deduce from (20) that a carrier schedules its service earlier (resp. later) than $T^*|_{k_0=k_1=0}$ when its marginal time cost is positive (resp. negative), and even earlier (resp. later) when its marginal time cost is large (resp. large in absolute value) or its equilibrium demand at its departure facility is low. Setting $F(T_0^*) = F(T_1^*)$, we can further characterize the principle of minimum differentiation in schedules. When the cost-demand ratio is equal for both carriers, i.e., when $k_0/D_0^*(\mathbf{T}^*, \boldsymbol{\tau}) = k_1/D_1^*(\mathbf{T}^*, \boldsymbol{\tau})$, the carriers schedule their service at the same time, earlier than $T^*|_{k_0=k_1=0}$ when $k_0, k_1 > 0$ and later than $T^*|_{k_0,k_1=0}$ when $k_0, k_1 < 0$. This situation could arise across rival carriers evenly sharing the market and facing the same marginal time cost ($k_0 = k_1 \neq 0$). *Aside from the case where carriers' marginal time cost is exactly proportional to their equilibrium demand*, we can infer from (20) that the existence of operational costs that vary with the time of day implies distinct service times in equilibrium.[33] The shape of $F(t)$ acts an additional agglomeration/deglomeration force driving service time locations closer/farther away from $T^*|_{k_0=k_1=0}$ on the time line. When the configuration of marginal time costs induces $T_0^* \geq T_1^*$, carrier 0 schedules its service later than carrier 1 and we need to rely on the expressions provided in Appendix C.4. Section 4.2 numerically explores all combinations of positive, negative and null marginal time costs across carriers leading to distinct service times when $F(t)$ is uniform. The following proposition summarizes our findings in the time game.

Proposition 3. *Consider two competing carriers ($i = 0, 1$) which set a single simultaneous departure time $T_i \in]0, 24[$ prior to setting fares. Assume that carriers' time cost is linear in the time of day, i.e., $K(T_i) = K_i + k_i T_i$ for $i = 0, 1$. Let β (γ) with $0 < \beta < \gamma$ be consumers' unit early (late) schedule delay costs. Then:*

1. if carriers' time cost is constant along the time of day, i.e., if $k_0 = k_1 = 0$, both carriers schedule their service at the $\beta/(\beta + \gamma)$th quantile of consumers' desired time distribution. As these optimal departure times are strictly dominant strategies for both carriers, the Nash equilibrium is unique and interior.

2. when carriers' time cost varies along the time of day, if a unique Nash equilibrium in departure times (denoted $\mathbf{T}^*$) exists, carriers schedule their service at the same time if $k_0/k_1 = D_0^*(\mathbf{T}^*, \boldsymbol{\tau})/D_1^*(\mathbf{T}^*, \boldsymbol{\tau})$ in (20) and at different times for all other marginal time costs and equilibrium demand configurations.

Proposition 3.2 stresses that when $k_i \neq 0$ for $i = 0, 1$, departure times are equal across carriers only when their marginal time cost is of same proportion as the equilibrium

[32] That is, $\mathrm{d}T_i^*/\mathrm{d}k_i < 0$ for $i = 0, 1$, see Appendix B.5.

[33] Appendix B.6 characterizes more formally the differences in departure times in terms of equilibrium demands and marginal time costs.

demand at their departure facility. To prove that Proposition 3 holds when $T_0 \geq T_1$, use the equilibrium demands (C.5) in the profit functions (15) and follow the same derivation steps (see Appendix C.4).

Borenstein and Netz (1999) and Salvanes et al. (2005) find empirical evidence that the principle of minimum differentiation in departure times applies in commercial aviation in deregulated markets. In our duopolistic model, the difference in schedules between rival carriers departing a different facility is small when consumers' desired time distribution is symmetric and tight on the time line and when carriers' marginal time cost is: (a) null for both carriers, (b) the same for both carriers *and* demand at their departure facility is close enough, (c) of same sign across carriers and proportional to the demand at their departure facility. Wider schedule differences arise in all other situations. Moreover, as noted in footnote 28, logistic constraints may induce the carriers to schedule their service at the boundaries of the time line. In our simple setup, several factors favour extreme service times in equilibrium. From the demand side, we can identify a strongly asymmetric schedule delay cost function and a large mass of consumers preferring early or late departure. Any structural parameter penalizing demand at a facility favours an extreme departure time for its carrier when $k_i \neq 0$. From the supply side, we can mention a large marginal time cost (in absolute value) along the day for a carrier.

Inserting the equilibrium service times $\mathbf{T}^*$ in demands (13) and in profits (15) yields:

$$D_0^*(\mathbf{T}^*, \boldsymbol{\tau}) = \frac{1}{6}\Big[3 + h + 2\Delta\widetilde{c} + 2\Phi(\mathbf{T}^*)\Big], \quad D_1^*(\mathbf{T}^*, \boldsymbol{\tau}) = 1 - D_0^*(\mathbf{T}^*, \boldsymbol{\tau}),$$
$$\pi_i^*(\mathbf{T}^*, \boldsymbol{\tau}) = D_i(\mathbf{T}^*, \boldsymbol{\tau})^2 - K(T_i^*), \quad i = 0, 1.$$

Having characterized carriers' market, we turn to the facility-rivalry game.

2.3 Facility-rivalry game

Consider two facilities, with fixed capacity, that compete with each other in the fees charged per passenger to their downstream firm. Assume that the facilities derive a per passenger commercial revenue from services provided by concessionaires to travellers. Without loss of generality, we set their marginal operational cost to zero and denote their fixed cost by $\widehat{F}_i \geq 0$ for $i = 0, 1$. So, facility maximization problem reads:

$$\max_{\widehat{\tau}_i} \widehat{\Pi}_i(\mathbf{T}, \widehat{\boldsymbol{\tau}}) = (\widehat{\tau}_i + \widehat{\omega}_i) D_i^*(\mathbf{T}, \boldsymbol{\tau}) - \widehat{F}_i \qquad i = 0, 1, \tag{24}$$

where $\widehat{\tau}_i \geq 0$ denotes the per passenger fee at facility i and $\widehat{\omega}_i \geq 0$ is the per passenger commercial revenue. In the last resolution step of the game, facilities take carrier fares and departure times $\mathbf{T}$ as given. Departure times can be either given to the carriers (as in Section 2.2.1) or optimally chosen (as in Section 2.2.2). In the latter case, when the time costs of a carrier vary with the time of day ($k_i \neq 0$) its optimal departure time in (20) is an implicit expression which depends on the (normalized) fees $\boldsymbol{\tau}$. Solving (24) analytically is not tractable. However, in all other cases, maximizing facility profits is straightforward and exactly follows the same resolution steps as carriers' fare game. If each facility simultaneously chooses its fee to maximize its profit and internalizes the pricing of its rival facility, solving the system of FOCs with respect to the fees yields:

$$\begin{aligned}\tau_0^* &= \frac{1}{3}\left[(c_1 - c_0) - (2\omega_0 + \omega_1)\right] + \frac{9+h}{6} + \frac{1}{3}\Phi(\mathbf{T}),\\ \tau_1^* &= \frac{1}{3}\left[(c_0 - c_1) - (\omega_0 + 2\omega_1)\right] + \frac{9-h}{6} - \frac{1}{3}\Phi(\mathbf{T}),\end{aligned} \tag{25}$$

where τ_i^* and ω_i are $\widehat{\tau}_i^*$ and $\widehat{\omega}_i$ divided by $\theta(1-h)$, and $\boldsymbol{\tau}^* \equiv (\tau_0^*, \tau_1^*)$ represents a Nash equilibrium in fees.[34] The optimal fee of a facility is decreasing in the marginal operational cost of its carrier and increasing in that of the carrier serving the rival facility. This result, identical to Basso and Zhang (2007), also stresses that a facility captures a fraction (1/3) of its carrier's operational cost advantage. Next, a higher per passenger revenue at *one* facility induces a lower fee at *both* facilities.[35] Notice that, in equilibrium, a facility shares part (2/3) of its own per passenger commercial revenue with its carrier and pushes the rival facility to reduce its fee by an amount that is proportional to its own per passenger revenue (1/3). The h term is the monopoly premium/penalty related to the location advantage/disadvantage of the facility and $\Phi(\mathbf{T})$ captures the SDC (dis)advantage due to potential differences in service times across carriers. Moreover, the facility fees capture the same share of the marginal operational and SDC advantages from the carriers.

Substituting the equilibrium fees (25) into demands (13) yields:

$$\begin{aligned}D_0^{f,*}(\mathbf{T}) &= \frac{1}{18}\left[9 + h + 2(c_1 - c_0) + 2(\omega_0 - \omega_1) + 2\Phi(\mathbf{T})\right],\\ D_1^{f,*}(\mathbf{T}) &= 1 - D_0^{f,*}(\mathbf{T}).\end{aligned} \tag{26}$$

Ultimately, the equilibrium demand for a facility decreases in the marginal operational cost of its carrier and in the per passenger commercial revenue of the rival facility; and increases in the marginal operational cost of the carrier serving the rival facility, in own per passenger commercial revenue and in the SDC advantage of its carrier. Note that, while per passenger commercial revenues help to keep the optimal fees low at both facilities, a per passenger commercial revenue advantage allows a facility to increase its demand at the expense of the rival facility.

[34] Again, the existence, uniqueness and local stability of the Nash equilibrium in fees are easy to demonstrate (see Vives, 1999, pp. 47-52).

[35] Substituting the optimal fees (25) in the equilibrium fares (8), we can establish how per passenger commercial revenues affect the equilibrium fares and carrier profits. Further replacing these fares in the covered market condition (2) allows to express the latter condition in terms of the exogenous parameters of the model. See Appendix B.7.

With the above equilibrium fees and demands at hand, facility profits in equilibrium are given by:

$$\Pi_i^*(\mathbf{T}) = 3D_i^{*2}(\mathbf{T}) - F_i, \quad i = 0, 1. \tag{27}$$

Similarly to the carrier market analysis, the differences in fees, demands and profits between facility 0 and facility 1 can be analyzed as follows:

$$\begin{aligned} \tau_0^* > \ \tau_1^* \quad & \textit{iff} \quad \Delta c \ > -\frac{h}{2} - \Phi(\mathbf{T}) + \frac{\Delta\omega}{2}, \\ D_0^{f,*}(\mathbf{T}) > \ D_1^{f,*}(\mathbf{T}) \quad & \textit{iff} \quad \Delta c \ > -\frac{h}{2} - \Phi(\mathbf{T}) - \Delta\omega, \\ \Pi_0^*(\mathbf{T}) > \ \Pi_1^*(\mathbf{T}) \quad & \textit{iff} \quad \Delta c \ > -\frac{h}{2} - \Phi(\mathbf{T}) - \frac{3}{2}\Delta F - \Delta\omega, \end{aligned} \tag{28}$$

where $\Delta c = c_1 - c_0$, $\Delta\omega = \omega_0 - \omega_1$, $\Delta F = F_1 - F_0$ with $F_i = \widehat{F}_i/\theta(1-h)$. The latter delta terms represent (normalized) costs or commercial revenue advantages for facility 0/facility 1 when they are positive/negative while $\Phi(\mathbf{T})$ captures the usual (normalized) SDC advantage. Setting $h = \Phi(\mathbf{T}) = \Delta\omega = \Delta F = 0$ in (28), in equilibrium, facility 0 charges a higher fee and receives a larger demand and profit than its rival when the marginal operational cost of its carrier is lower than that of the carrier serving the rival facility. Introducing a location (or a SDC or a fixed-cost) advantage for facility 0, a facility can charge a higher fee than its rival and get higher demand and profit even if the marginal operational cost of its carrier is larger than that of the carrier serving the rival facility. This happens when the marginal operational cost disadvantage of its carrier is not below $-\frac{h}{2}$ (or below $-\Phi(\mathbf{T})$ or below $-\frac{3}{2}\Delta F$, respectively). Setting $h = \Phi(\mathbf{T}) = \Delta F = 0$ in (28), we can focus on the effect of the per passenger commercial revenue advantage on the differences in fees, demands and profits across facilities. We deduce that a facility charges a higher fee than the rival facility if the marginal operational cost advantage of its carrier with respect to the carrier serving the rival facility outweighs half of its per passenger commercial revenue advantage and a facility faces a larger demand and receives a larger profit than the rival facility if the marginal operational cost *disadvantage* of its carrier with respect to the carrier serving the rival facility offsets its per passenger commercial revenue advantage. The following proposition summarizes the results of the facility-rivalry subgame.

Proposition 4. *Consider two competing facilities (i=0,1) that set the fees charged to their carrier with constant time costs along the time of day. Further assume that the carriers schedule their service such that $T_0 \leq T_1$. Then:*

1. there exists a unique Nash Equilibrium in fees given by (25)*,*

2. in equilibrium, demands and profits are given by (26) *and* (27)*, respectively. Facility 0 charges a higher fee, faces a larger demand and receives a larger profit than its rival when the three inequalities in* (28) *hold.*

To prove that Proposition 4 also holds when $T_0 \geq T_1$, use the equilibrium demands (C.5) in the profit functions (24) and follow the same steps. The SDC difference $\Phi(\mathbf{T})$ in the related expressions will be replaced by its counterpart $\Phi^{sym}(\mathbf{T})$ given in Eq. (C.3).

Having characterized the duopolistic outcome of the three-stage game, we now explore regulator's problem.

3 Regulator's problem

We now determine the location of the facilities and departure times that minimize the social costs (or, equivalently, maximize the social welfare). In the transportation industry, the location of the facilities is often decided by regional authorities in order to minimize consumer access costs. By contrast, schedule decisions often require broad coordination between a variety of agents (carriers, facility managers, regional and national authorities, international transport regulator) and may need to obey international standards. In air transportation, local and national entities decide where to locate the airports, while IATA provides the air transport community a single set of rules for the management of airport slots worldwide when the available infrastructure is insufficient to satisfy airline demands. Hence, we treat the location and schedule decisions independently of one another.[36]

Choosing the geographical location of firms/facilities that minimizes the average total transportation cost is a standard social planner problem in Hotelling's model. It amounts to minimizing the area below the transportation cost functions in Fig. 2. Assuming equal marginal operational costs across carriers and equal marginal commercial revenues across facilities, the optimal location of facility 0 and facility 1 can be easily shown to be at coordinates $(1/4, 3/4)$ when they are simultaneously chosen, at $1/3$ for facility 0 when facility 1 is located at 1, and at $(2+h)/3$ for facility 1 when facility 0 is located at h with $0 \leq h < 1$.

For the schedule regulator, minimizing the average total time cost of society (denoted $\widehat{SC}^S$ below), ignoring geographic locations, requires finding the departure time of each carrier that minimizes the area under consumer schedule delay cost functions in Fig. 1 plus carriers' total time cost in (18), that is:

$$\min_{T_0,T_1} \widehat{SC}^S = \widehat{SDC}^S(\mathbf{T}) + \widehat{K}^S(\mathbf{T}), \tag{29}$$

[36] The scaling factor $1/\theta(1-h)$ is not needed in this section; hence, all 'hat' magnitudes represent unscaled monetary values.

where

$$\widehat{SDC}^S(\mathbf{T}) = \int_0^{T_0} \widehat{\gamma}(T_0 - t)\rho(t)\,\mathrm{d}t + \int_{T_0}^{\widetilde{t}} \widehat{\beta}(t - T_0)\rho(t)\,\mathrm{d}t + \int_{\widetilde{t}}^{T_1} \widehat{\gamma}(T_1 - t)\rho(t)\,\mathrm{d}t + \int_{T_1}^{24} \widehat{\beta}(t - T_1)\rho(t)\,\mathrm{d}t, \tag{30}$$

$$\widehat{K}^S(\mathbf{T}) = \widehat{K}_0 + \widehat{k}_0 T_0 + \widehat{K}_1 + \widehat{k}_1 T_1$$

and where $T_0 \leq T_1$.[37] Term $\widetilde{t} = (\widehat{\beta}T_0 + \widehat{\gamma}T_1)/(\widehat{\beta} + \widehat{\gamma})$ in (30) is the abscissa of the intersection between the dotted and the bold schedule delay cost functions in Fig. 1. Using the same tools as in Appendix B.3, the FOCs of (29) are given by:[38]

$$\frac{\partial \widehat{SC}^S(\mathbf{T})}{\partial T_0} = \widehat{\gamma} m_\ell - \widetilde{m}_\ell \widehat{\beta} + \widehat{k}_0 = 0, \quad \frac{\partial \widehat{SC}^S(\mathbf{T})}{\partial T_1} = \widetilde{m}_r \widehat{\gamma} - \widehat{\beta} m_r + \widehat{k}_1 = 0, \tag{31}$$

where terms $\widetilde{m}_\ell = \int_{T_0}^{\widetilde{t}} \rho(t)dt$ and $\widetilde{m}_r = \int_{\widetilde{t}}^{T_1} \rho(t)dt$ have now a familiar interpretation. Using $\rho(t) = \mathcal{U}[0, 24]$ for analytical tractability, the $\widehat{SC}^S$ function is given by:

$$\widehat{SC}^S = \frac{\widehat{\gamma}(2\widehat{\beta} + \widehat{\gamma})T_0^2 + \widehat{\beta}(\widehat{\beta} + 2\widehat{\gamma})T_1^2 - 48\widehat{\beta}(\widehat{\beta} + \widehat{\gamma})T_1 - 2\widehat{\beta}\widehat{\gamma}T_0T_1 + 576\widehat{\beta}(\widehat{\beta} + \widehat{\gamma})}{48(\widehat{\beta} + \widehat{\gamma})} + \widehat{K}^S(\mathbf{T}).$$

Solving the FOCs, we obtain the socially optimal departure times:

$$T_0^S = \frac{12\widehat{\beta}}{\widehat{\beta} + \widehat{\gamma}} - a_0\widehat{k}_0 - b_0\widehat{k}_1, \quad T_1^S = 12\left[1 + \frac{\widehat{\beta}}{\widehat{\beta} + \widehat{\gamma}}\right] - a_1\widehat{k}_0 - b_1\widehat{k}_1, \tag{32}$$

where $a_i, b_i > 0$ for $i = 0, 1$.[39] Assuming symmetric unit schedule delay costs and null marginal time costs ($\widehat{\beta} = \widehat{\gamma}$ and $\widehat{k}_0 = \widehat{k}_1 = 0$) in (32), the social planner sets $T_0^S = 6$ (at the first quartile of the $\mathcal{U}[0, 24]$ distribution) and $T_1^S = 18$ (at the third quartile) − the classical result of Hotelling's simultaneous location-game with symmetric transportation costs−. Focusing on the case where $\widehat{k}_0 = \widehat{k}_1 = 0$ with $\widehat{\beta} < \widehat{\gamma}$, denoting the related socially optimal service times by $T_i^S|_{\widehat{k}_0=\widehat{k}_1=0}$ for $i = 0, 1$ and by using the duopolistic equilibrium departure times in (22) with the uniform distribution, we deduce that:

$$\Delta T_0 = T_0^S|_{\widehat{k}_0=\widehat{k}_1=0} - T^*|_{\widehat{k}_0=\widehat{k}_1=0} = -12\frac{\widehat{\beta}}{\widehat{\beta} + \widehat{\gamma}},$$

$$\Delta T_1 = T_1^S|_{\widehat{k}_0=\widehat{k}_1=0} - T^*|_{\widehat{k}_0=\widehat{k}_1=0} = 12\frac{\widehat{\gamma}}{\widehat{\beta} + \widehat{\gamma}}.$$

[37]The case $T_0 \geq T_1$ is derived in Appendix C.5.

[38]These derivations are lengthy but straightforward. Sufficient conditions for (29) are provided in Appendix B.8.

[39]To obtain T_1^S in (32), note that $12\left[1 + \frac{\widehat{\beta}}{\widehat{\beta}+\widehat{\gamma}}\right] \equiv 12\left[\frac{2\widehat{\beta}}{\widehat{\beta}+\widehat{\gamma}} + \frac{\widehat{\gamma}}{\widehat{\beta}+\widehat{\gamma}}\right]$. Moreover, the a_i and b_i terms are given by $a_0 = \frac{12(\widehat{\beta}+2\widehat{\gamma})}{\widehat{\gamma}(\widehat{\beta}+\widehat{\gamma})} > 0$, $b_0 = a_1 = \frac{12}{\widehat{\beta}+\widehat{\gamma}} > 0$ and $b_1 = \frac{12(2\widehat{\beta}+\widehat{\gamma})}{\widehat{\beta}(\widehat{\beta}+\widehat{\gamma})} > 0$.

Hence, duopolistic competition results in service times which are later/earlier than socially optimal at facility 0/facility 1 when the marginal time costs are null across carriers. The schedule regulator could either impose the socially optimal service times or equalize the marginal social time costs with carriers' marginal time costs. Using T_0^S and T_1^S in (20), we get an explicit optimal pricing rule for the service times that reads:

$$\widehat{k}_0^S = \frac{\widehat{\beta} D_0^S}{3}, \quad \widehat{k}_1^S = -\frac{\widehat{\gamma} D_1^S}{3}, \tag{33}$$

where $D_i^S \equiv D_i^*(\mathbf{T}^S, \boldsymbol{\tau})$ and $\mathbf{T}^S \equiv (T_0^S|_{\widehat{k}_0=\widehat{k}_1=0}, T_1^S|_{\widehat{k}_0=\widehat{k}_1=0})$. The above result indicates that the schedule regulator should set an increasing cost in the time of day for the carrier serving facility 0 to get the early socially optimal service time and a decreasing one for the carrier operating at facility 1 to get the late socially optimal departure time. The following proposition summarizes the above results.

Proposition 5. *Consider a regulator that chooses the departure time of the carrier that operates at each facility (T_0^S and T_1^S with $T_0^S \leq T_1^S$) over the* $[0, 24]$ *time interval to minimize the average total time cost* (29)*. Assume a uniform distribution of consumers' desired departure time. Then:*

1. if the time costs of the carriers are constant along the time of day, the socially optimal service times are such that $T_0^S < T^*|_{\widehat{k}_0=\widehat{k}_1=0} < T_1^S$,

2. if the time costs of the carriers vary along the time of day, the socially optimal service time of the carriers is given by (32)*.*

Further substituting (33) in (20) and solving for T_i, the social and private marginal time costs coincide and each carrier schedules a socially optimal departure time. The distributional impacts of an optimal location of facility 0 and of a socially optimal schedule for both carriers are briefly analyzed below.

4 Numerical results

This section illustrates our analytical results. Section 4.1 focuses on the three-stage game assuming that carriers' time cost *does not vary* with the time of day. In this case, a unique Nash equilibrium is guaranteed in each subgame and can be calculated with the closed-form expressions given in the theoretical sections. Section 4.2 concentrates on the carrier-rivalry game with given fees and time-varying operational costs for the carriers. In this case, the Nash equilibria are established numerically. In all simulations, we use the "hat notation" and express the cost parameters, fares, fees and profits in unscaled monetary units, i.e., not divided by $\theta(1-h)$. We posit a unit transportation cost of $\theta/2 = \$130$ which implies an inter-facility transportation cost of \$73 when facility 0 is located at $h = 0.25$ and facility 1 is at $x = 1$. Unit schedule delay costs are set to $\widehat{\beta} = \$5$ and $\widehat{\gamma} = \$7$ and we assume an uniform distribution $\mathcal{U}[0, 24]$ of consumers' desired departure time.[40] We posit a higher marginal operational cost for the carrier serving facility 0 ($\widehat{c}_0 = \$10$ and $\widehat{c}_1 = \$8$) and a higher per passenger commercial revenue for facility 0 ($\widehat{\omega}_0 = \$20$ and $\widehat{\omega}_1 = \$18$). One can think of carrier 0 as a legacy carrier operating at a primary facility that competes with a lower marginal operational cost carrier serving a secondary facility with a lower per passenger commercial revenue. Without loss of generality, we set all fixed costs to zero ($\widehat{K}_i = \widehat{F}_i = 0$ for $i = 0, 1$).

4.1 Equilibria with constant time costs across carriers

Table 1 shows the equilibria of the three-stage game assuming a null marginal time cost ($\widehat{k}_0 = \widehat{k}_1 = 0$) for each carrier.[41] By Proposition 3.1, duopolistic competition drives to an identical departure time across carriers and to a travel service scheduled in the morning, at the quantile $\widehat{\beta}/(\widehat{\beta} + \widehat{\gamma}) = 5/(5 + 7) = 41.\bar{6}\%$ of the $\mathcal{U}[0, 24]$ distribution. Thus, $T_0^* = T_1^* = 41.\bar{6}\% \times 24 = 10$. This schedule strategy is dominant for both carriers: given a suboptimal time for a carrier, its rival at the other facility has no incentive to change its service time.

The first column of results (referred to as Column 1) in Table 1 illustrates the market equilibrium when we combine minimum schedule differentiation (resulting from the above equilibrium departure times) with maximum spatial differentiation. Then, all else equal, we successively introduce a location advantage for facility 0 in Column 2 and a small/large SDC disadvantage for its carrier in Column 3/4. Column 5 compares the equilibria of Columns 1 to 4 with those arising when a regulator sets the location of facility 0 and carriers' departure time to minimize the social costs, as discussed in Section 3. Carriers' marginal time cost being null here, the social time cost equals consumers' average total schedule delay cost. Recall that the three-stage game presumes that the departure times are *simultaneously* set for both carriers (endogenously or exogenously).

Clearly, Column 1 leads to the highest fares, average transportation and time costs for the consumers, the highest fees for the carriers and the highest profits for the facilities and their carrier. Note that, in equilibrium, the fees differ across facilities but the fares, demands, and profits are equal. This result may seem surprising at first glance but it is fully consistent with our main propositions.[42]

[40] These unit schedule delay costs' values are slightly lower than those used in Van der Weijde et al. (2014) but closer to the willingness to pay found by Brey and Walker (2011) for air travellers. Also, recall that when t is uniform, $\widehat{\Phi}(\mathbf{T}) \equiv \widehat{\Phi}^{sym}(\mathbf{T})$. Thus, the price, demand and profit functions of each carrier and facility remain identical whether $T_0 \leq T_1$ or $T_0 \geq T_1$.

[41] For each column of Table 1, the reader can verify that the covered market condition (2) and all theoretical results hold. In particular, all differences in equilibrium prices, demands and profits between facilities and between carriers can be analyzed in light of our propositions.

[42] Propositions 1, 2 and 4.2 are particularly easy to verify in the context of Column 1. By way

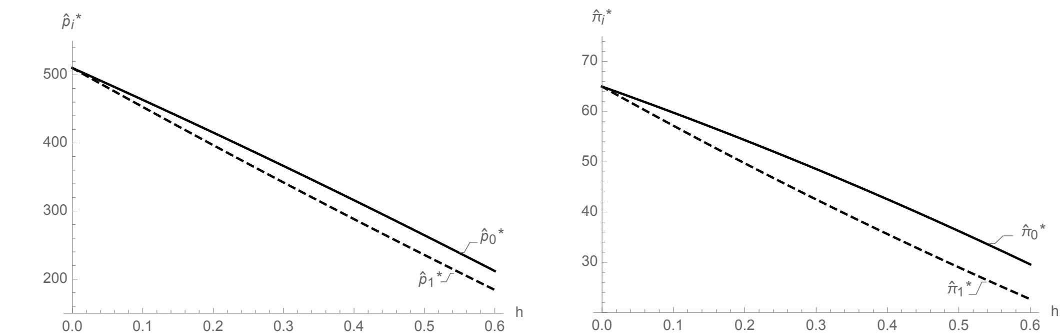
Table 1: Market and regulated equilibria with null marginal costs

	$T_0^* = T_1^* = 10$ $h = 0$ $\widehat{\Phi} = 0$	$T_0^* = T_1^* = 10$ $h = 0.25$ $\widehat{\Phi} = 0$	$T_0 = 7$; $T_1^* = 10$ $h = 0.25$ $\widehat{\Phi} = -2.25$	$T_0 = 0$; $T_1^* = 10$ $h = 0.25$ $\widehat{\Phi} = -25$	$T_0^S = 5$; $T_1^S = 17$ $h^S = 0.33$ $\widehat{\Phi} = 6$
Facilities					
$(\widehat{\tau}_0^*; \widehat{\tau}_1^*)$	$(370, 372)$	$(280.6, 266.4)$	$(279.9, 267.1)$	$(272.3, 274.7)$	$(252.9, 231.7)$
$(\widehat{\Pi}_0^*, \widehat{\Pi}_1^*)$	$(195, 195)$	$(154.5, 138.2)$	$(153.7, 139.0)$	$(146.0, 146.5)$	$(142.5, 119.3)$
Carriers					
$(\widehat{p}_0^*, \widehat{p}_1^*)$	$(510, 510)$	$(390.8, 369.2)$	$(389.8, 370.2)$	$(379.7, 380.3)$	$(353.8, 323)$
$(\widehat{\pi}_0^*, \widehat{\pi}_1^*)$	$(65, 65)$	$(51.5, 46.1)$	$(51.2, 46.3)$	$(48.7, 48.8)$	$(47.5, 39.8)$
Consumers					
$(D_0^*; D_1^*)$	$(0.5, 0.5)$	$(0.514, 0.486)$	$(0.513, 0.487)$	$(0.499, 0.501)$	$(0.522, 0.478)$
Schedule delay cost†	35	35	28.11	26.49	17.5
Transportation cost†	10.83	5.25	5.25	5.25	4.82
Total Cost	45.83	40.25	33.36	31.74	22.32

Notes: All simulations assume that $t \sim \mathcal{U}[0, 24]$, $\widehat{\beta} = 5, \widehat{\gamma} = 7$, $\frac{\theta}{2} = 130$, $\widehat{c}_0 = 10$, $\widehat{c}_1 = 8$, $\widehat{k}_0 = \widehat{k}_1 = 0$, $\widehat{\omega}_0 = 20$, $\widehat{\omega}_1 = 18$, $\widehat{K}_i = \widehat{F}_i = 0$ for $i = 0, 1$. †The average total schedule delay and transportation costs of consumers are computed from a regulator's perspective.

In Column 2, we introduce a location advantage for facility 0 that represents 25% of the total market size. As compared to Column 1, consumers' average total transportation cost is lower from a regulator standpoint (75% of the travellers are now closer to facility 0) and the average total schedule delay cost remains the same as service times are not affected. Carrier fares and facility fees decrease in h due to the rise in competition while consumers' demand at facility 0/facility 1 raises/falls. The profits of the facilities and their carrier decrease along h. This is expected at facility 1 as the optimal fare, fee and demand all drop when h increases. At facility 0, the decrease in profits stresses that the competition effect dominates the demand effect. Note that introducing a location advantage that benefits facility 0 and its carrier hurts more the profit of their rivals (-7.8% for facility 0 and its carrier and -13.8% for their rivals). Fig. 6 confirms the latter results[43] and shows that the fare and profit of carrier 0 are

Figure 6: Carrier fares, profits as a function of h when service times are equal



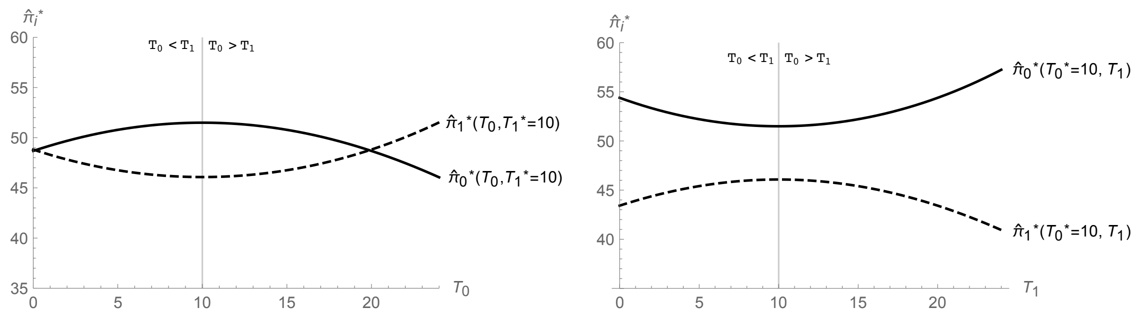
of example, note that $\Delta\widehat{c} = -\Delta\widehat{\omega} < -\Delta\widehat{\omega}/2$. Proposition 4.2 implies $D_0^{f,*} = D_1^{f,*}$, $\widehat{\Pi}_0^* = \widehat{\Pi}_1^*$ and $\widehat{\tau}_0^* < \widehat{\tau}_1^*$, which corresponds to the simulation results.

[43] Facility plots are not shown due to space constraints but they are similar to carrier plots (up to a vertical shift).

always above those of its rival along h. This need not always be the case (see below).

Fig. 7 illustrates how a deviation from the optimal departure time for *one* carrier affects carrier profits. The LHS plot shows the profits of both carriers for $T_0 \in [0, 24]$ when $T_1^* = 10$. Carrier 0's profit decreases for any deviation from T_0^* while carrier 1's profit increases. Note that a deviation of 10 hours to the LHS or RHS of $T_0^* = 10$ would allow carrier 1 to make a higher profit than its rival despite carrier 0's location advantage. Taking the perspective of carrier 1 on the RHS plot in Fig. 7, carrier 1's profit decreases as T_1 deviates from $T_1^* = 10$ while carrier 0's profit increases.

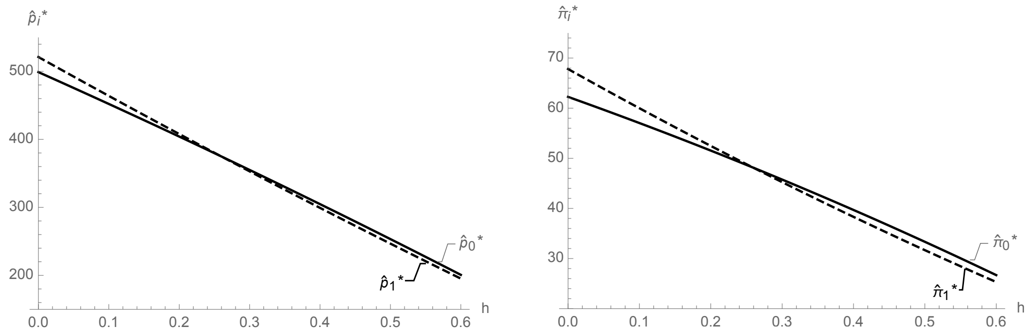
Figure 7: Carrier profits as a function of departure times



In Column 3 (resp. 4), carrier 0 is given a departure time 3 (resp. 10) hours earlier than its optimal departure time which induces a "small" (resp. "large") SDC disadvantage for that carrier (and for its departing facility). As compared to Column 2, the average total schedule delay cost is lower from a regulator viewpoint (the departure times are now different across facilities) without affecting consumers' transportation costs. The equilibrium demand, prices and profits at facility 0 all fall and those at facility 1 all rise. These results are related to Lemma 2 through Eqs. (11) and (16). In equilibrium, any deviation from the optimal departure time for a carrier decreases its fare, demand and profit as well as the fee, demand and profit of its departure facility and benefits the rival carrier and facility. Note that Fig. 6 remains valid in the context of Column 3: the competition effect dominates the demand effect along h and the profit of carrier 0 remains above that of carrier 1 when h increases. However, in the context of Column 4, demand, prices and profits at facility 0 fall slightly below those at facility 1. Fig. 8 shows that the competition effect dominates the demand effect in Column 4 as well, but the location advantage must be large enough (slightly above 25%) for the profits and prices at facility 0 to be larger than those at facility 1. This underlines the importance of the spatial advantage for the profitability of a facility and a (less cost-effective) carrier when the departure times are not optimal.

Turning to the regulated framework of Section 3, the optimal location of facility 0 on the geographic space (given facility 1 located at 1) is at 1/3 and the socially optimal departure times, given by (32), are $T_0^S = 5$ and $T_1^S = 17$. As expected, consumers' total costs are the lowest from the regulator standpoint in Column 5 of Table 1. Demand

Figure 8: Carrier fares, profits as a function of h when departure times are strongly differentiated



at facility 0/facility 1 increases/decreases as its location and SDC advantages/disadvantages are now larger. The fees, fares and profits all decrease.

4.2 Equilibria with time-varying costs across carriers

This section focuses on the carrier-rivalry game and investigates all marginal time cost pairs of the set $\widehat{k}_0 \times \widehat{k}_1$ with $\widehat{k}_i = \{-1, 0, 1\}$ for $i = 0, 1$ that lead to distinct (profit-maximizing) departure times.[44] To further highlight demand's role on schedule differentiation, we also study the equilibria obtained assuming $\widehat{k}_0 = \widehat{k}_1 > 0$, $\widehat{k}_0 = \widehat{k}_1 < 0$ and $\widehat{k}_i = -\widehat{k}_{-i} > 0$ for $i = 0, 1$. Facility fees are set to $\widehat{\tau}_0 = \$280$, $\widehat{\tau}_1 = \$266$ (as in Column 2 of Table 1) and we assume that both facilities operate 24h/day.

Fig. 9 shows the best response functions of each carrier to their rival's schedule, denoted $BR_i(\widehat{k}_i)$, $i = 0, 1$, for the above combinations of marginal time costs.[45] The "NE" intersections are Nash equilibria in departure times such that $T_0^* < T_1^*$ ($T_0^* > T_1^*$) when they are *above* (*below*) the $T_0 = T_1$ diagonal. Table A.1 in Appendix A provides the numerical values of the departure times, fares, demands and profits related to Fig. 9, along with equilibria obtained with alternative unit schedule delay costs.

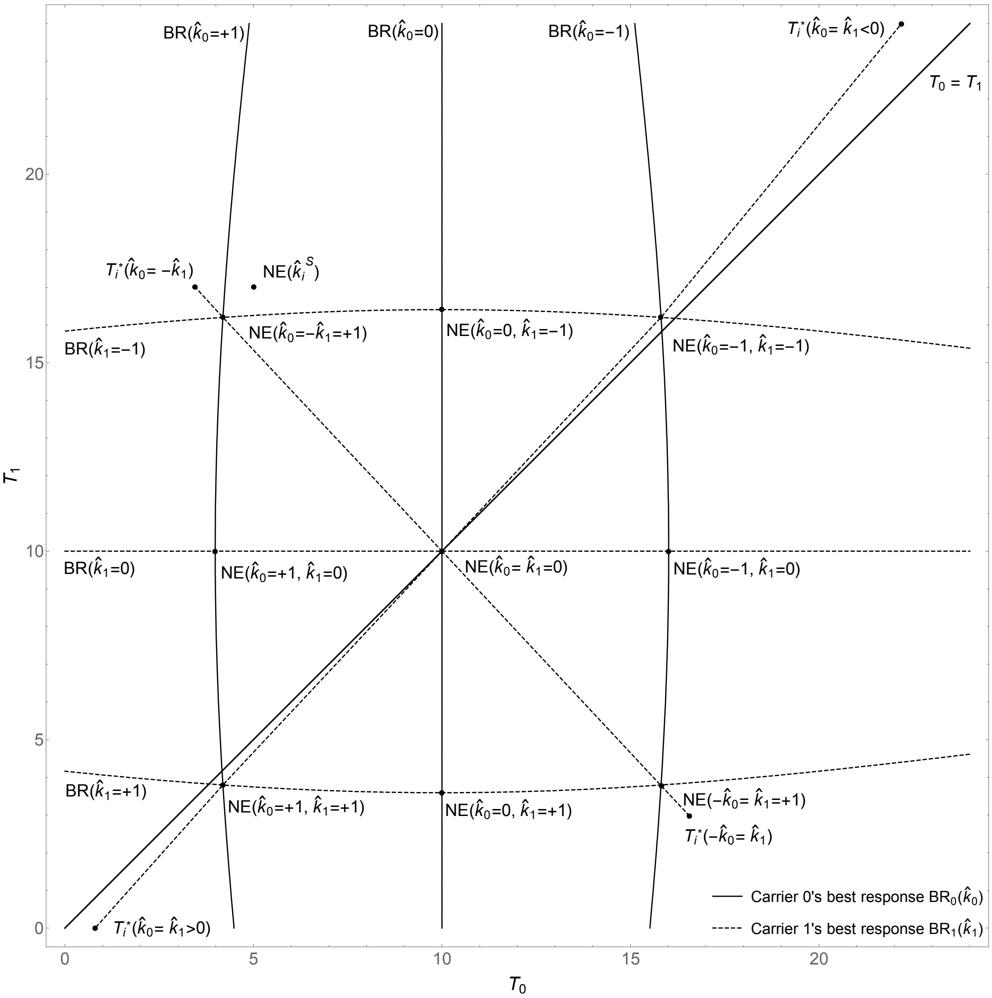
When the marginal time cost is null for both carriers, their reaction functions $BR_0(\widehat{k}_0 = 0)$ and $BR_1(\widehat{k}_1 = 0)$ are constant and intersect at point $NE(\widehat{k}_0 = \widehat{k}_1 = 0)=$ $(10, 10)$. When the time cost increases in the time of day for carrier 0, its reaction function, denoted $BR_0(\widehat{k}_0 = +1)$, moves leftwards and becomes convex in the departure time of its rival. The intersection between $BR_0(\widehat{k}_0 = +1)$ and $BR_1(\widehat{k}_1 = 0)$ yields a unique Nash equilibrium $NE(\widehat{k}_0 = +1, \widehat{k}_1 = 0) = (4, 10)$. Hence, carrier 0 schedules

[44] The reader can check that the existence conditions, multiplied by the positive scaling factor $\theta(1-h)$, hold. That is (B.11) yields $-241.9 < \Delta\widehat{\widehat{c}} = -16 < 193.13$, (B.12) leads to $-2.01 < \widehat{k}_0 < 1.57$, $-1.88 < \widehat{k}_1 < 1.48$ and (B.13) is given by $0 < 5 < 195/24 = 8.125$ and $7 > 0$.

[45] Fare reaction functions are not plotted here but they are as expected (linearly increasing in the fare of the rival carrier and intersecting on $\mathbb{R}_+^2$).

its service earlier in the morning while carrier 1 departs at 10. The remaining Nash equilibria can be analyzed similarly.

Figure 9: Nash Equilibria in service time and reaction functions for the carrier-rivalry game



Notes: all simulations assume: $h = 0.25$, $t \sim \mathcal{U}[0, 24]$, $\widehat{\beta} = 5, \widehat{\gamma} = 7$, $\frac{\theta}{2} = 130$, $\widehat{c}_0 = 10$, $\widehat{c}_1 = 8$, $\widehat{\tau}_0 = 280$, $\widehat{\tau}_1 = 266$, $\widehat{K}_i = 0$ for $i = 0, 1$.

Consider the case where the marginal time cost is *the same* across carriers and different from zero $(\widehat{k}_0 = \widehat{k}_1 \neq 0)$. These equilibria lie along the dotted lines $T_i^*(\widehat{k}_0 = \widehat{k}_1 < 0)$ and $T_i^*(\widehat{k}_0 = \widehat{k}_1 > 0)$, close to the $T_0 = T_1$ diagonal. Both services are scheduled *later/earlier* than 10 when the time costs *decrease/increase* in the time of day. As carrier 1 always receives a *lower* demand in equilibrium (see Table A.1), its departure time moves farther away to the RHS of 10 than that of its rival when the marginal time costs are negative (then, $T_0^* < T_1^*$), and farther away to the LHS of 10 when the marginal time costs are positive (then, $T_0^* > T_1^*$). The proximity of the lines $T_i^*(\widehat{k}_0 = \widehat{k}_1 < 0)$ and $T_i^*(\widehat{k}_0 = \widehat{k}_1 > 0)$ to the diagonal $T_0 = T_1$ is due to the closeness of the equilibrium demand of the carriers. By contrast, when the marginal time cost is of same magnitude across carriers but of *different sign*, larger (and asymmetric) differentiation in schedules occurs, see the dotted lines $T_i^*(\widehat{k}_0 = -\widehat{k}_1)$ and $T_i^*(-\widehat{k}_0 = \widehat{k}_1)$.[46]

Finally, notice in Fig. 9 that any small deviation from the equilibrium for a carrier along its reaction function drives its departure time back to the initial equilibrium. When a carrier schedules its service slightly earlier/later than optimal, its rival responds by scheduling its service slightly earlier/later as well. This situation being suboptimal for both carriers, the subsequent time adjustments push the schedules back to the initial Nash equilibrium. This confirms the stability in schedule competition of the carrier-rivalry game and stresses that service times are strategic complements.

All the above reasonings remain valid when consumers' unit schedule delay costs are symmetric or when the postulated asymmetry is reversed. Table A.1 in Appendix A numerically illustrates these two cases for $\widehat{\beta} = \widehat{\gamma} = 7$ and when $\widehat{\beta} = 7 > \widehat{\gamma} = 5$. Assuming a null marginal time cost for both carriers, the former case leads to services scheduled at the median consumers' desired time, at 12, while the latter one yields services scheduled in the afternoon, at 14. All the other results are as expected.

5 Conclusion

This paper proposes a framework to analyze the rivalry in prices and in service times between facilities, for the case in which the facilities provide an input to downstream firms that sell the final service to consumers (vertical structure) at a specific time of day. The model allows (i) asymmetries in the location of the facilities along the geographic space and in consumers' valuation of schedule delays (schedule delay costs), (ii) a general distribution of consumers' desired service time and (iii) heterogeneous operational costs related to the timing of the service along the time of day for the downstream firms. This setup is used to analyze the competition between a primary airport and its carrier, conveniently located in a linear city, and a rival secondary airport and its carrier located in a remote place (the extremity) of the city. We assume that each carrier schedules a single flight toward the same destination, operates alone at its airport and sets a single departure time.

We find that accounting for the costs incurred by the carriers in the timing of

[46] When $\widehat{k}_0 = -\widehat{k}_1 = 1.13$, then $T_0^* = 3.4$, $T_1^* = 17$, $\widehat{p}_0^* = 390.8$ and $\widehat{p}_1^* = 368.2$, and the larger term within inequality (2) is close to the upper bound. Similarly, when $-\widehat{k}_0 = \widehat{k}_1 = 1.13$, then $T_0^* = 16.6$, $T_1^* = 3$, $\widehat{p}_0^* = 390.8$ and $\widehat{p}_1^* = 368.2$, and the lower term within inequality (C.4) is close to its lower bound. Thus, $T_i^*(\widehat{k}_0 = -\widehat{k}_1)$ and $T_i^*(-\widehat{k}_0 = \widehat{k}_1)$ are bounded within the $[0, 24]^2$ space to ensure a covered market.

the service is essential for identifying the level of differentiation in departure times in duopolistic airline markets: (*i*) when the operational cost of the carriers does not vary with the time of day, the travel service is offered at the same time, (*ii*) when this cost varies with the time of day and is *identical across carriers*, differences in equilibrium demands across facilities suffice to generate (moderately) distinct departure times, (*iii*) when this cost varies with the time of day *and differs across carriers*, the travel service is generally scheduled at a different time by each carrier and the level of schedule differentiation is proportional to the marginal time cost faced by the carriers and to facility demands. By letting the distribution of consumers' departure time unspecified, we show explicitly how this distribution interacts with consumers' valuation of schedule delays and carriers' marginal time cost to determine the optimal service times, fares, fees, demands and profits at each airport.

The paper also identifies all price markups of the vertical structure. In equilibrium, we establish that a higher per passenger commercial revenue at *one* airport induces a lower per passenger fee charged by *both* airports to their carrier and a lower fare charged by *both* carriers at their departure airport. A lower marginal operational cost for *one* carrier implies a higher fee charged at its departure airport, a lower fee at the other airport served by the rival carrier and a lower fare at *both* airports. When an airport is more conveniently located for travellers, it can set a higher fee and its downstream carrier can charge a higher fare. Differentiation in departure times allows the airport and its carrier to compete in an additional differentiation dimension that can reduce or strengthen the advantage in location.

This model could be extended in a number of directions. Stackelberg games would clearly refine our results regarding strategic behaviours. Considering heterogeneous transportation costs toward the facilities would allow to better characterize the role played by the location advantage. Allowing a wider range of departure times in the spirit of Lindsey and Tomaszewska (1999) would help to design realistic schedule policies to improve the social welfare. Future research may want to consider price-elastic demands for the individual consumers as in Van der Weijde et al. (2014) and to conduct simulations based on more realistic distributions of travellers' desired service time as our simulations focus on the uniform shape.

Acknowledgments

Preliminary versions of this paper were presented at the ITEA Annual Conference 2016 in Santiago (Chile), at the 56th SCSE Annual Congress 2016, in Québec (Canada) and at the PhD Workshop organized by CREATE in 2015 at Laval University. We also benefited from helpful comments from Simon Anderson, Achim Czerny, Gilles B. Koumou, Koami A. Dzigbodi, Julien Monardo and Cliff Winston. Laingo Manitra Randrianarisoa would like to thank the members of her PhD committee (Philippe

Barla, Vincent Boucher, Michel Roland, Christos Constantatos) and Markus Herrmann for their detailed feedback. Financial help from the Airport Research Chair of Laval University is gratefully acknowledged. The usual disclaimer applies.

References

Adler, T., C. Falzarano, and G. Spitz, "Modelling service trade-offs in air itinerary choices," *Transportation Research Record*, January 2005, *1915*, 20–26.

Anderson, S. P., "Equilibrium existence in the linear model of spatial competition," *Economica*, 1988, *55* (220), 479–491.

_ , K. Goeree, and R. Ramer, "Location, Location Location," *Journal of Economic Theory*, 1997, *77*, 102–127.

Arnott, Richard, A. De Palma, and R. Lindsey, "A structural model of peak-period congestion: A traffic bottleneck with elastic demand," *American Economic Review*, March 1993, *83* (1), 161–179.

Basso, L. J and A. Zhang, "Congestible facility rivalry in vertical structures," *Journal of Urban Economics*, 2007, *61* (2), 218–237.

Ben-Akiva, M., A. De Palma, and J.-F. Thisse, "Spatial competition with differentiated products," *Regional Science and Urban Economics*, February 1989, *19* (1), 5–19.

Bley, K. and T. Buermann, "Business processes and IT solutions in the low fare environment," in S. Gross and A. Schroeder, eds., *Handbook of Low Cost Airlines. Strategies, Business Processes and Market Environment*, Erich Schmidt Verlag, 2007, chapter 1, pp. 55–76. Partially available on Google Books: URL: `https://books.google.com`.

Borenstein, S. and J. Netz, "Why do all the flights leave at 8 am?: Competition and departure-time differentiation in airline markets," *International Journal of Industrial Organization*, 1999, *17* (5), 611–640.

Brey, R. and J. L. Walker, "Latent temporal preferences: An application to airline travel," *Transportation Research Part A: Policy and Practice*, 2011, *45* (9), 880–895.

Brueckner, J. K., "Airport congestion when carriers have market power," *American Economic Review*, 2002, pp. 1357–1375.

_ , "Airport congestion management: prices or quantities?," *ACCESS*, fall 2009, (35), 1–6.

___ , "Schedule competition revisited," *Journal of Transport Economics and Policy*, 2010, *44* (3), 261–285.

___ **and R. Flores-Fillol**, "Airline schedule competition," *Review of Industrial Organization*, 2007, *30* (3), 161–177.

D'Aspremont, C., J. J. Gabszewicz, and J-F. Thisse, "On Hotelling's stability in competition," *Econometrica: Journal of the Econometric Society*, 1979, pp. 1145–1150.

De Borger, B. and K. Van Dender, "Prices, capacities and service levels in a congestible Bertrand duopoly," *Journal of Urban Economics*, 2006, *60* (2), 264–283.

___ **and S. Proost**, "Transport policy competition between governments: A selective survey of the literature," *Economics of Transportation*, 2012, *1*, 35–48.

___ **, F. Dunkerley, and S. Proost**, "Strategic investment and pricing decisions in a congested transport corridor," *Journal of Urban Economics*, 2007, *62*, 294–316.

De Palma, A. and L. Leruth, "Congestion and game in capacity: a duopoly analysis in the presence of network externalities," *Annales d'Économie et de Statistique*, 1989, pp. 389–407.

___ **, V. Ginsburgh, Y. Y. Papageorgiou, and J-F. Thisse**, "The principle of minimum differentiation holds under sufficient heterogeneity," *Econometrica: Journal of the Econometric Society*, 1985, pp. 767–781.

Done, K., "Continental pays Heathrow record," News article, The Financial Times 2008. URL: `http://www.ft.com/cms/s/0/b6a47274-e955-11dc-8365-0000779fd2ac.html`.

Douglas, G. W. and J. C. Miller, "Economic regulation of domestic air transport: theory and policy," Technical Report, Brookings Institution, Washington, D.C., 1974.

Encaoua, D., M. Moreaux, and A. Perrot, "Compatibility and competition in airlines: demand side network effects," *International Journal of Industrial Organization*, 1996, (14), 701–26.

Etschmaier, M. and D. Mathaisel, "Airline scheduling: An overview," *Transportation Science*, May 1985, *19* (2), 127–138.

Gillen, D., "Airports slots: A primer," in A. I. Czerny, P. Forsyth, David Gillen, and H.M. Niemeier, eds., *Airports slots. International experiences and options for reform*, Ashagate Publishing Limited, 2008, chapter 4, pp. 1–432.

Greater Toronto Airports Authority, "Aeronautical Charges and Fees," Official webpage, Toronto Pearson International 2016. URL: `https://www.torontopearson.com/en/Airport_Charges_and_Fees/#`.

Gross, S. and A. Schroeder, "Basic Business Model of European Low Cost Airlines. An Analysis," in S. Gross and A. Schroeder, eds., *Handbook of Low Cost Airlines. Strategies, Business Processes and Market Environment*, Erich Schmidt Verlag, 2007, chapter 1, pp. 31–50.

Hotelling, H., "Stability in competition," *The Economic Journal*, 1929, *39* (153), 41–57.

IATA, "Airline cost performance," Economics Briefing 5, International Air Transport Association July 2005. URL: `https://www.iata.org/whatwedo/Documents/economics/airline_cost_performance.pdf`.

Koster, P., E. Pels, and E. Verhoef, "The User Costs of Air Travel Delay Variability," *Transportation Science*, 2014, pp. 120–131.

Lijesen, M. G., "A mixed logit based valuation of frequency in civil aviation from SP-data," *Transportation Research Part E: Logistics and Transportation Review*, 2006, *42* (2), 82–94.

Lindsey, R. and E. Tomaszewska, *Schedule competition, fare competition and predation in a duopoly airline market*, Oum, T. and B. Bowen ed., Proceedings of the Air Transportation Group (ATRG), Institute of Aviation, University of Nebraska at Omaha, 1999.

Oum, T. H., A. Zhang, and Y. Zhang, "Alternative forms of economic regulation and their efficiency implications for airports," *Journal of Transport Economics and Policy*, 2004, *38* (2), 217–246.

Panzar, J. C., "Equilibrium and welfare in unregulated airline markets," *The American Economic Review*, 1979, pp. 92–95.

Pels, E. and E. T. Verhoef, "The economics of airport congestion pricing," *Journal of Urban Economics*, 2004, *55* (2), 257–277.

_ , P. Nijkamp, and P. Rietveld, "Airport and airline competition for passengers departing from a large metropolitan area," *Journal of Urban Economics*, 2000, *48* (1), 29–45.

_ , _ , and _ , "Access to and competition between airports: a case study for the San Francisco Bay area," *Transportation Research Part A: Policy and Practice*, 2003, *37* (1), 71–83.

Salvanes, K. G., F. Steen, and L. Sørgard, "Hotelling in the air? Flight departures in Norway," *Regional Science and Urban Economics*, 2005, *35* (2), 193–213.

Small, K. A., "The scheduling of consumer activities: work trips," *The American Economic Review*, 1982, *72* (3), 467–479.

Ulrich, C., "How the present (IATA) slot allocation works," in A. I. Czerny, P. Forsyth, David Gillen, and H.M. Niemeier, eds., *Airports slots. International experiences and options for reform*, Ashagate Publishing Limited, 2008, chapter 2, pp. 1–432.

Van Dender, K., "Duopoly prices under congested access," *Journal of Regional Science*, 2005, *45* (2), 343–362.

Van der Weijde, A. H., E. T. Verhoef, and V. A. C. Van den Berg, "A Hotelling Model with Price-sensitive Demand and Asymetric Distance Costs," *Journal of Transport Economics and Policy*, May 2014, *48* (2), 261–277.

Verhoef, E. T., "Congestion pricing, slots sales and slot trading in aviation," *Transportation Research Part B: Methodological*, 2010, (44), 320–329.

Vickrey, W. S., "Congestion theory and transport investment," *The American Economic Review*, 1969, pp. 251–260.

Vives, Xavier, *Oligopoly Pricing. Old ideas and new tools.*, MIT Press, 1999.

Warburg, V., C. Bhat, and T. Adler, "Modeling demographic and unobserved heterogeneity in air passengers' sensitivity to service attributes in itinerary choice," *Transportation Research Record: Journal of the Transportation Research Board*, 2006, (1951), 7–16.

Zhang, A. and A. I. Czerny, "Airports and airlines economics and policy: An interpretive review of recent research," *Economics of Transportation*, 2012, *1* (1), 15–34.

_ **and Y. Zhang**, "Concession revenue and optimal airport pricing," *Transportation Research Part E: Logistics and Transportation Review*, 1997, *33* (4), 287–296.

_ **and** _ , "Airport charges and capacity expansion: effects of concessions and privatization," *Journal of Urban Economics*, 2003, *53*, 54–75.

A Table A.1 of Section 4.2

Table A.1: Carrier-rivalry equilibria with time-varying marginal costs for the carriers

	$T_0 \le T_1$				
	Carrier marginal time costs				
	$\widehat{k}_0 = 0$, $\widehat{k}_1 = 0$	$\widehat{k}_0 = 0$, $\widehat{k}_1 = -1$	$\widehat{k}_0 = +1$, $\widehat{k}_1 = 0$	$\widehat{k}_0 = +1$, $\widehat{k}_1 = -1$	$\widehat{k}_0 = -1$, $\widehat{k}_1 = -1$
			$\widehat{\beta} = 5$, $\widehat{\gamma} = 7$		
(T_0^*, T_1^*)	$(10, 10)$	$(10, 16.4)$	$(4, 10)$	$(4.2, 16.2)$	$(15.8, 16.2)$
$(\widehat{p}_0^*, \widehat{p}_1^*)$	$(390.3, 368.7)$	$(393.7, 365.3)$	$(387.3, 371.7)$	$(390.7, 368.3)$	$(390.7, 368.3)$
(D_0^*, D_1^*)	$(0.514, 0.486)$	$(0.532, 0.468)$	$(0.499, 0.501)$	$(0.516, 0.484)$	$(0.516, 0.484)$
$(\widehat{\pi}_0^*, \widehat{\pi}_1^*)$	$(51.6, 46.0)$	$(55.2, 59.1)$	$(44.5, 49.0)$	$(47.8, 61.8)$	$(67.8, 61.8)$
			$\widehat{\beta} = 7$, $\widehat{\gamma} = 7$		
(T_0^*, T_1^*)	$(12, 12)$	$(12, 17.5)$	$(6.9, 12)$	$(7, 17.3)$	$(17.0, 17.3)$
$(\widehat{p}_0^*, \widehat{p}_1^*)$	$(390.3, 368.7)$	$(393.2, 365.8)$	$(387.7, 371.3)$	$(390.6, 368.4)$	$(390.6, 368.4)$
(D_0^*, D_1^*)	$(0.514, 0.486)$	$(0.529, 0.471)$	$(0.501, 0.499)$	$(0.516, 0.484)$	$(0.516, 0.484)$
$(\widehat{\pi}_0^*, \widehat{\pi}_1^*)$	$(51.6, 46.0)$	$(54.6, 60.7)$	$(42.1, 48.5)$	$(44.9, 63.0)$	$(68.9, 63.0)$
			$\widehat{\beta} = 7$, $\widehat{\gamma} = 5$		
(T_0^*, T_1^*)	$(14, 14)$	$(14, 20.4)$	$(8, 14)$	$(8.2, 20.2)$	$(19.8, 20.2)$
$(\widehat{p}_0^*, \widehat{p}_1^*)$	$(390.3, 368.7)$	$(393.7, 365.3)$	$(387.3, 371.7)$	$(390.7, 368.3)$	$(390.7, 368.3)$
(D_0^*, D_1^*)	$(0.514, 0.486)$	$(0.532, 0.468)$	$(0.499, 0.501)$	$(0.516, 0.484)$	$(0.516, 0.484)$
$(\widehat{\pi}_0^*, \widehat{\pi}_1^*)$	$(51.6, 46.0)$	$(55.2, 63.1)$	$(40.5, 49.0)$	$(43.8, 65.8)$	$(71.8, 65.8)$
	$T_0 \ge T_1^\dagger$				
	Carrier marginal time costs				
	$\widehat{k}_0 = 0$, $\widehat{k}_1 = 0$	$\widehat{k}_0 = 0$, $\widehat{k}_1 = +1$	$\widehat{k}_0 = -1$, $\widehat{k}_1 = 0$	$\widehat{k}_0 = -1$, $\widehat{k}_1 = +1$	$\widehat{k}_0 = +1$, $\widehat{k}_1 = +1$
			$\widehat{\beta} = 5$, $\widehat{\gamma} = 7$		
$(T_0^{sym,*}, T_1^{sym,*})$	$(10, 10)$	$(10, 3.6)$	$(16, 10)$	$(15.8, 3.8)$	$(4.2, 3.8)$
$(\widehat{\pi}_0^{sym,*}, \widehat{\pi}_1^{sym,*})$	$(51.6, 46.0)$	$(55.2, 39.1)$	$(64.5, 49.0)$	$(67.8, 41.8)$	$(47.8, 41.8)$
			$\widehat{\beta} = 7$, $\widehat{\gamma} = 7$		
(T_0^*, T_1^*)	$(12, 12)$	$(12, 6.5)$	$(17.1, 12)$	$(17, 6.7)$	$(7, 6.7)$
$(\widehat{\pi}_0^{sym,*}, \widehat{\pi}_1^{sym,*})$	$(51.6, 46.0)$	$(54.6, 36.7)$	$(66.1, 48.5)$	$(68.9, 39.0)$	$(44.9, 39.0)$
			$\widehat{\beta} = 7$, $\widehat{\gamma} = 5$		
$(T_0^{sym,*}, T_1^{sym,*})$	$(14, 14)$	$(14, 7.6)$	$(20, 14)$	$(19.8, 7.8)$	$(8.2, 7.8)$
$(\widehat{\pi}_0^{sym,*}, \widehat{\pi}_1^{sym,*})$	$(51.6, 46.0)$	$(55.2, 35.1)$	$(49.0, 68.5)$	$(71.8, 37.8)$	$(43.8, 37.8)$

Notes: all simulations assume: $h = 0.25$, $t \sim \mathcal{U}[0, 24]$, $\frac{\theta}{2} = 130$, $\widehat{c}_0 = 10$, $\widehat{c}_1 = 8$, $\widehat{\tau}_0 = 280$, $\widehat{\tau}_1 = 266$, $\widehat{K}_i = 0$ for $i = 0, 1$. † The equilibrium fares $(\widehat{p}_0^*, \widehat{p}_1^*)$ and demands (D_0^*, D_1^*) when $T_0 \ge T_1$ are identical to those reported in the same column for $T_0 \le T_1$ due to $\Phi^U(\mathbf{T}) \equiv \Phi^{sym,U}(\mathbf{T})$.

B Derivations assuming $T_0 \leq T_1$

B.1 Demands (3)

The net benefit of travelling from each facility for a consumer located at $x \in [0,1]$ with desired time $\underline{t \in [0, T_0]}$ is given by:

$$\begin{aligned}
\widehat{U}_0^\ell &= \widehat{U} - \widehat{p}_0 - \frac{\theta}{2}(x-h)^2 - \widehat{\gamma}(T_0 - t), \\
\widehat{U}_1^\ell &= \widehat{U} - \widehat{p}_1 - \frac{\theta}{2}(1-x)^2 - \widehat{\gamma}(T_1 - t),
\end{aligned}$$

where superscript ℓ designates magnitudes related to consumers with t at the LHS of T_0. Solving $\widehat{U}_0^\ell - \widehat{U}_1^\ell$ for x leads to the indifferent consumer:

$$\tilde{x}^\ell = \frac{1}{\theta(1-h)}[\widehat{p}_1 - \widehat{p}_0 + \widehat{\gamma}(T_1 - T_0)] + \frac{1+h}{2}. \tag{B.1}$$

As consumers are uniformly distributed with density one ($f(x) = 1$) on $x \in [0,1]$, consume a single unit of the good, and given a distribution $\rho(t)$ of desired departure times, market demands for those with $t \in [0, T_0]$ are given by:

$$\begin{aligned}
D_0^\ell &= \int_0^{T_0} \int_0^{\tilde{x}^\ell} f(x)\rho(t)\,\mathrm{d}x\mathrm{d}t = \tilde{x}^\ell \int_0^{T_0} \rho(t)\,\mathrm{d}t \\
&= \left[p_1 - p_0 + \gamma(T_1 - T_0) + \frac{1+h}{2}\right] m_\ell, \qquad\qquad (\text{B.2}) \\
D_1^\ell &= m_\ell - D_0^\ell = \left[p_0 - p_1 - \gamma(T_1 - T_0) + \frac{1-h}{2}\right] m_\ell,
\end{aligned}$$

where m_ℓ denotes the share of consumers with $t \in [0, T_0]$ as defined under (4). Following the same reasoning for the consumers with $\underline{t \in]T_0, T_1[}$, we get:

$$\begin{aligned}
\widehat{U}_0^c &= \widehat{U} - \widehat{p}_0 - \frac{\theta}{2}(x-h)^2 - \widehat{\beta}(t - T_0), \\
\widehat{U}_1^c &= \widehat{U} - \widehat{p}_1 - \frac{\theta}{2}(1-x)^2 - \widehat{\gamma}(T_1 - t), \\
\tilde{x}^c(t) &= \frac{1}{\theta(1-h)}(\widehat{p}_1 - \widehat{p}_0) + \frac{\widehat{\beta}T_0 + \widehat{\gamma}T_1 - (\widehat{\gamma} + \widehat{\beta})t}{\theta(1-h)} + \frac{(1+h)}{2}; \qquad (\text{B.3})
\end{aligned}$$

$$\begin{aligned}
D_0^c &= \int_{T_0}^{T_1} \tilde{x}^c(t)\rho(t)\,\mathrm{d}t = \left[p_1 - p_0 + (\gamma T_1 + \beta T_0) + \frac{1+h}{2}\right] m_c - (\beta + \gamma)\bar{t}_c, \qquad (\text{B.4}) \\
D_1^c &= m_c - D_0^c = \left[p_0 - p_1 - (\gamma T_1 + \beta T_0) + \frac{1-h}{2}\right] m_c + (\beta + \gamma)\bar{t}_c,
\end{aligned}$$

where superscript c designates magnitudes related to consumers with t between T_0 and T_1, m_c is the share of consumers with $t \in]T_0, T_1[$ and $\bar{t}_c$ is the expected desired departure time defined under (4). Turning to those with $t \in [T_1, 24]$, we get:

$$\widehat{U}_0^r = \widehat{U} - \widehat{p}_0 - \frac{\theta}{2}(x-h)^2 - \widehat{\beta}(t - T_0),$$
$$\widehat{U}_1^r = \widehat{U} - \widehat{p}_1 - \frac{\theta}{2}(1-x)^2 - \widehat{\beta}(t - T_1),$$
$$\tilde{x}^r = \frac{1}{\theta(1-h)}[\widehat{p}_1 - \widehat{p}_0 - \widehat{\beta}(T_1 - T_0)] + \frac{(1+h)}{2}; \tag{B.5}$$
$$D_0^r = \tilde{x}^r \int_{T_1}^{24} \rho(t)\,\mathrm{d}t = \left[p_1 - p_0 - \beta(T_1 - T_0) + \frac{1+h}{2}\right] m_r, \tag{B.6}$$
$$D_1^r = \left[p_0 - p_1 + \beta(T_1 - T_0) + \frac{1-h}{2}\right] m_r,$$

where superscript r designates magnitudes related to consumers with t at the RHS of T_1 and m_r is the share of consumers with $t \in [T_1, 24]$ as defined under (4). Aggregating the demands over the entire [0,24] segment − setting $D_0 = D_0^\ell + D_0^c + D_0^r$, using (B.2), (B.4) and (B.6) and recalling that $m_\ell + m_c + m_r = 1$ − we get D_0 in (3). Further using the covered condition, we get $D_1 = 1 - D_0$.

Focusing on the SDC difference term (4), by collecting the T_0 and T_1 terms and dispatching the average central terms $\bar{t}_c$, we obtain:

$$\Phi(\mathbf{T}) = \underbrace{(\gamma m_\ell + \gamma m_c - \beta m_r)T_1 - \gamma \bar{t}_c}_{C_1^M} - \underbrace{[(\gamma m_\ell - \beta m_c - \beta m_r)T_0 + \beta \bar{t}_c]}_{C_0^M},$$

where C_i^M is the average *normalized* schedule delay cost difference related to each departure time T_i for $i = 0, 1$. Then, given that $\Phi(\mathbf{T}) = \left[\widehat{C}_1^M - \widehat{C}_0^M\right]/\theta(1-h)$, by using (3), setting $\partial D_0/\partial \eta > 0$ with $\eta = \{h, \theta\}$ and rearranging, we get:

$$\begin{aligned} &\frac{\partial D_0(\mathbf{p}, \mathbf{T})}{\partial h} > 0 \quad \textit{iff} \quad \frac{\theta}{2}(1-h)^2 > (\widehat{p}_0 + \widehat{C}_0^M) - (\widehat{p}_1 + \widehat{C}_1^M), \\ &\frac{\partial D_0(\mathbf{p}, \mathbf{T})}{\partial \theta} > 0 \quad \textit{iff} \quad \widehat{p}_1 + \widehat{C}_1^M > \widehat{p}_0 + \widehat{C}_0^M, \end{aligned} \tag{B.7}$$

where $(\widehat{p}_0 + \widehat{C}_0^M) - (\widehat{p}_1 + \widehat{C}_1^M) \equiv -(\widehat{p}_1 - \widehat{p}_0) - \widehat{\Phi}(\mathbf{T})$ and $\partial D_1(\mathbf{p}, \mathbf{T})/\partial \eta = -\partial D_0(\mathbf{p}, \mathbf{T})/\partial \eta$. The link between (B.7) and the results under (1) should be clear.

B.2 Covered market condition (2)

To establish market conditions which guarantee that $0 < \widetilde{x}(t) < 1$ for $t \in [0, 24]$, use (B.1), (B.3) and (B.5) and note that $\tilde{x}^\ell(t) \leq \tilde{x}^c(t) \leq \tilde{x}^r(t)$. Then, setting $0 < \tilde{x}^\ell(t) \leq \tilde{x}^r(t) < 1$ yields (2).

B.3 Proof of Lemma 2

To compute $\partial\Phi(\mathbf{T})/\partial T_i$ for $i = 0, 1$, note first that:

$$\frac{\partial m_l}{\partial T_0} = \rho(T_0), \quad \frac{\partial m_c}{\partial T_0} = -\rho(T_0), \quad \frac{\partial m_r}{\partial T_0} = 0, \quad \frac{\partial \bar{t}_c}{\partial T_0} = -T_0\rho(T_0),$$
$$\frac{\partial m_l}{\partial T_1} = 0, \quad \frac{\partial m_c}{\partial T_1} = \rho(T_1), \quad \frac{\partial m_r}{\partial T_1} = -\rho(T_1), \quad \frac{\partial \bar{t}_c}{\partial T_1} = T_1\rho(T_1).$$

Using the above, we obtain:

$$\frac{\partial\Phi(\mathbf{T})}{\partial T_0} = \beta - (\beta+\gamma)m_\ell, \qquad \frac{\partial\Phi(\mathbf{T})}{\partial T_1} = \gamma - (\beta+\gamma)m_r, \tag{B.8}$$
$$\frac{\partial^2\Phi(\mathbf{T})}{\partial T_0^2} = -(\beta+\gamma)\rho(T_0), \qquad \frac{\partial^2\Phi(\mathbf{T})}{\partial T_1^2} = (\beta+\gamma)\rho(T_1), \tag{B.9}$$

where we use $m_c + m_r = 1 - m_\ell$ to get $\partial\Phi(\mathbf{T})/\partial T_0$ and $m_\ell + m_c = 1 - m_r$ to get $\partial\Phi(\mathbf{T})/\partial T_1$. Setting each first derivative in (B.8) equal to zero leads to the departure times $T_0^* = T_1^* = F^{-1}[\beta/(\beta+\gamma)]$, where we use $\gamma/(\beta+\gamma) = 1 - \beta/(\beta+\gamma)$ to obtain T_1^*.

B.4 Existence of a schedule equilibrium when $\rho(t) = \mathcal{U}[0, 24]$

When $\rho(t) = \mathcal{U}[0, 24]$, by using $\Phi^U(T_0, T_1)$ in (5) and demands (13) in carrier profits (15), these profits can be shown to be two quartic functions in T_i that reads:

$$\pi_i(T_0, T_1) = f_i + e_iT_i + d_iT_i^2 + bT_i^3 + aT_i^4, \quad i = 0, 1, \tag{B.10}$$

where coefficients $a = (\beta+\gamma)^2/20736$ and $b = -\beta(\beta+\gamma)/216$ are identical across carriers and depend only on the (normalized) unit schedule delay costs, coefficients d_i, e_i and f_i are carrier-specific and depend on T_{-i} and other parameters of the model.[47] Let $T_i, T_{-i} \in \mathbb{R}$. The coefficient of leading term T_i^4 being positive ($a > 0$), we deduce that $\lim_{T_i\to\pm\infty} \pi_i(T_i, T_{-i}) = +\infty$. For a unique and interior profit-maximizing departure time $T_i^\star \in]0, 24[$ to exist for $i = 0, 1$, the profit functions need to be locally concave on $[0, 24]$. They must be 'W-shaped' and thus possess three critical points, two of which are minima and one is a local maximum. Below we provide a set of parameter restrictions which ensure that this local maximum exists, is unique and interior on the

[47] The detailed expressions of this Appendix are available upon request.

[0,24] time interval and yields positive profits for $i = 0, 1$. To establish this, we impose

$$
\begin{aligned}
&(i) \quad \frac{\partial^2 \pi_i(T_0, T_1)}{\partial T_i^2} < 0, \ \forall T_i \in \left]T_{i,a}^{\star\star}, T_{i,b}^{\star\star}\right[\subset \mathbb{R} \text{ with } T_i = 0 \in \left]T_{i,a}^{\star\star}, T_{i,b}^{\star\star}\right[,\\
&(ii) \quad \left.\frac{\partial \pi_i(T_0, T_1)}{\partial T_i}\right|_{T_i=0} > 0, \ \left.\frac{\partial \pi_i(T_0, T_1)}{\partial T_i}\right|_{T_i=24} < 0 \quad \text{and}\\
&(iii) \quad \pi_i(T_0, T_1)|_{T_i=0} > 0
\end{aligned}
$$

for $i = 0, 1$. Condition (i) ensures the existence of a locally quasi-concave profit function for each carrier over the real interval $[T_{i,a}^{\star\star}, T_{i,b}^{\star\star}] \subset \mathbb{R}$ which includes $T_i = 0$. On this interval, the *first derivative* of the profit function is decreasing. Condition (ii) guarantees that this first derivative is: positive at $T_i = 0$, null at the local maximum $T_i^\star$ and negative on $]T_i^\star, 24]$. This necessarily implies the existence of a single stationary point $T_i^\star \in]0, 24[$ which is the profit-maximizing departure time for each carrier. Condition (iii) makes sure that the maximized profit for $i = 0, 1$ is positive because $\pi_i(T_0, T_1)|_{T_i=T_i^\star}$ is above $\pi_i(T_0, T_1)|_{T_i=0} > 0$.

Starting with (i), by (B.10) we have:

$$\partial^2 \pi_i / \partial T_i^2 = \bar{d}_i + \bar{b} T_i + \bar{a} T_i^2, \quad i = 0, 1,$$

where the coefficient of the leading terms T_i^2 is positive ($\bar{a} = (\beta + \gamma)^2/1728 > 0$). We need parameter restrictions which ensure the existence of two real solutions (or roots) to $\partial^2 \pi_i(T_0, T_1)/\partial T_i^2 = 0$, denoted $T_{i,a}^{\star\star}$ and $T_{i,b}^{\star\star}$ and such that $T_{i,a}^{\star\star} < T_{i,b}^{\star\star}$ and $\partial^2 \pi_i / \partial T_i^2 < 0$, $\forall T_i \in]T_{i,a}^{\star\star}, T_{i,b}^{\star\star}[$. These roots are the inflection points of the quartic profit functions. A quadratic function has two real solutions if its discriminant satisfies $\Delta_i = \bar{b}^2 - 4\bar{a}\bar{d}_i > 0$. As $\bar{a}, \bar{b}^2 > 0$, we can impose $\bar{d}_i < 0$. Term $\bar{d}_i$ can be shown to be itself a quadratic function in T_{-i} such that $\bar{d}_i = \bar{\bar{d}}_i + \bar{\bar{b}} T_{-i} + \bar{\bar{a}} T_{-i}^2$ with $\bar{\bar{a}} = -(\beta + \gamma)/5184 < 0$. Let the discriminant of $\bar{d}_i$ be given by $\Delta_{\bar{d}_i}$. Setting $\Delta_{\bar{d}_i} < 0$ for $i = 0, 1$ guarantees that $sgn(\bar{d}_i) = sgn(\bar{\bar{a}})$ and implies $\bar{d}_i < 0, \forall T_{-i} \in \mathbb{R}$. This restriction leads to the following first condition:

$$
\begin{gathered}
\beta, \gamma > 0, \quad 0 \le h < 1, \quad T_0, T_1 \in \mathbb{R} \quad \text{and}\\
\frac{72\beta^2 - (\beta + \gamma)(3 + h)}{2(\beta + \gamma)} < \Delta\widetilde{c} < \frac{-72\beta^2 + (\beta + \gamma)(3 - h)}{2(\beta + \gamma)}.
\end{gathered}
\tag{B.11}
$$

Next, solving $\partial^2 \pi_i(T_0, T_1)/\partial T_i^2 = 0$ for $i = 0, 1$ yields:

$$T_{0,a,b}^{\star\star} = \frac{24\beta}{\beta + \gamma} \mp \frac{\sqrt{A}}{\sqrt{3}(\beta + \gamma)}, \quad T_{1,a,b}^{\star\star} = \frac{24\beta}{\beta + \gamma} \mp \frac{\sqrt{A'}}{\sqrt{3}(\beta + \gamma)}$$

where $T_{i,a}^{\star\star} < T_{i,b}^{\star\star}$ because

$$
\begin{aligned}
A &= 24\left[24\beta^2 + (3 + h + 2\Delta\widetilde{c})(\beta + \gamma)\right] - 48\beta(\beta + \gamma)T_1 + (\beta + \gamma)^2 T_1^2 > 0,\\
A' &= 24\left[24\beta^2 + (3 - h - 2\Delta\widetilde{c})(\beta + \gamma)\right] - 48\beta(\beta + \gamma)T_0 + (\beta + \gamma)^2 T_0^2 > 0.
\end{aligned}
$$

We can further show that $T^{\star\star}_{i,a} < 0$ and $T^{\star\star}_{i,b} > 0$ when (B.11) holds so that $T_i = 0 \in]T^{\star\star}_{i,a}, T^{\star\star}_{i,b}[$ for $i = 0, 1$.

For satisfying (ii), note first that $\partial\pi_i(T_0, T_1)/\partial T_i|_{T_i=0} = e_i$ in (B.10) and we can show that $e_i = d'_i + b'T_{-i} + a'T^2_{-i}$ with $a' = \beta(\beta+\gamma)/216 > 0$. Then, the first derivative of the profit function evaluated at $T_i = 0$ for $i = 0, 1$ appears to be, again, a quadratic function in T_{-i} and the coefficient of the leading term T^2_{-i} − identical across carriers − is positive. To ensure that $e_i > 0$, $\forall T_{-i} \in \mathbb{R}$, we require its discriminant to be lower than zero so that $sgn(e_i) = sgn(a')$. The same reasoning applies to $\partial\pi_i(T_0, T_1)/\partial T_i|_{T_i=24} < 0$. In the latter case, the coefficient of the leading term T^2_{-i} can be shown to be negative ($a' = -\gamma(\beta+\gamma)/216 < 0$) and so must be set the related discriminant. Combining the resulting restrictions for $i = 0, 1$ yields to the following set of conditions:

$$\begin{aligned} \frac{8\gamma^3}{3(\beta+\gamma)} - \frac{(3+h+2\Delta\widetilde{c})\gamma}{9} < k_0 < -\frac{8\beta^3}{3(\beta+\gamma)} + \frac{(3+h+2\Delta\widetilde{c})\beta}{9}, \\ \frac{8\gamma^3}{3(\beta+\gamma)} - \frac{(3-h-2\Delta\widetilde{c})\gamma}{9} < k_1 < -\frac{8\beta^3}{3(\beta+\gamma)} + \frac{(3-h-2\Delta\widetilde{c})\beta}{9}. \end{aligned} \tag{B.12}$$

Turning to (iii), $\pi_i(T_0, T_1)|_{T_i=0}$ is clearly equivalent to f_i in (B.10), where f_i can be shown to be a fourth-degree polynomial in T_{-i} which does not depend on k_i. Further setting the fixed time costs $K_i = 0$ for simplicity, when (B.11) holds, we can show that $f_i > 0$ if:[48]

$$0 < \beta < \frac{1}{24} \quad \text{and} \quad \gamma > 0 \qquad \text{or when} \qquad \beta > \frac{1}{24} \quad \text{and} \quad \gamma > \beta(24\beta - 1). \tag{B.13}$$

Replacing $\beta, \gamma > 0$ in (B.11) by (B.13) guarantees that $\pi_i(T_0, T_1)|_{T_i=0} > 0$ for $i = 0, 1$.

To sum up, (B.11), (B.12) and (B.13) ensure the existence a unique profit-maximizing $T^\star_i \in]0, 24[$ for each carrier with positive profits. Notice that these conditions are expressed in terms of scaled parameters. In the simulations of Section 4.2 (see footnote 44 in particular) we multiply all existence conditions by $\theta(1-h)$ to get their unscaled counterpart. Of course, these interior solutions do not guarantee the *uniqueness of the Nash equilibrium in departure times*. These additional conditions are excessively technical. Alternatively, one can simply use the parameter values which satisfy the existence conditions, solve system $\partial\pi_i(T_0, T_1)/\partial T_i = 0$ for $i = 0, 1$ and draw the time reaction functions that fall within the $[0, 24]^2$ space (one per carrier). This approach is more insightful and allows to graphically explore the uniqueness of the Nash equilibrium and its local stability. This is the path we take in the simulations of Section 4.2.

[48] This result is obtained with Mathematica after solving: `Reduce`[$f_i > 0$ && (B.11)]. Plotting f_i for $i = 0, 1$ under the above parameter restrictions yields a quartic function which is positive $\forall T_{-i} \in \mathbb{R}$.

B.5 Differentiation of the FOCs (20) w.r.t. k_i

To show that $\mathrm{d}T_i^*/\mathrm{d}k_i < 0$, rearrange first (20) as

$$G(T_i, k_i) \equiv F(T_i) + \frac{3k_i}{2(\beta+\gamma)D_i^*(\mathbf{T}, \boldsymbol{\tau})} - \frac{\beta}{\beta+\gamma} = 0.$$

The implicit function theorem implies that $\mathrm{d}T_i/\mathrm{d}k_i = -(G_{k_i}/G_{T_i})$. After some algebra, we get:

$$\frac{\mathrm{d}T_i}{\mathrm{d}k_i} \equiv -\frac{G_{k_i}}{G_{T_i}} = \frac{3D_i^*(\mathbf{T}, \boldsymbol{\tau})}{3k_i \frac{\partial D_i^*(\mathbf{T}, \boldsymbol{\tau})}{\partial T_i} - \rho(T_i)2(\beta+\gamma)D_i^{*,2}(\mathbf{T}, \boldsymbol{\tau})}. \tag{B.14}$$

Next, we deduced from (13) that $\partial D_i^*(\mathbf{T}, \boldsymbol{\tau})/\partial T_i = \frac{1}{3}\Phi_{T_i}(\mathbf{T})$ for $i = 0, 1$ and by Lemma 2 we have $\Phi_{T_0}(\mathbf{T}) = \beta - (\beta+\gamma)m_\ell$ and $\Phi_{T_1}(\mathbf{T}) = \gamma - (\beta+\gamma)m_r$. Assuming the existence of an interior and unique Nash equilibrium $\mathbf{T}^* = (T_0^*, T_1^*)$, FOCs (19) satisfy

$$\beta - (\beta+\gamma)m_\ell = \frac{3k_0}{2D_0^*(\mathbf{T}^*, \boldsymbol{\tau})} \quad \text{and} \quad \gamma - (\beta+\gamma)m_r = \frac{3k_1}{2D_1^*(\mathbf{T}^*.\boldsymbol{\tau})}, \tag{B.15}$$

Thus, $\partial D_i^*(\mathbf{T}, \boldsymbol{\tau})/\partial T_i = \frac{1}{3}\Phi_{T_i}(\mathbf{T})$ becomes $\partial D_i^*(\mathbf{T}^*, \boldsymbol{\tau})/\partial T_i = k_i/2D_i^*(\mathbf{T}^*, \boldsymbol{\tau})$. Replacing the latter expressions in (B.14) for $i = 0, 1$, the denominator of (B.14) becomes:

$$\frac{3k_i^2}{2D_i^*(\mathbf{T}^*, \boldsymbol{\tau})} - \rho(T_i^*)2(\beta+\gamma)D_i^{*,2}(\mathbf{T}^*, \boldsymbol{\tau}), \quad i = 0, 1. \tag{B.16}$$

Assuming that (23) is satisfied in equilibrium for $i = 0, 1$, the above expression is negative. That is, setting (B.16) < 0 yields (23). Thus, given that the numerator of (B.14) is positive and its denominator is negative in equilibrium, we deduce that $\mathrm{d}T_i^*/\mathrm{d}k_i < 0$.

B.6 Schedule differentiation with time-varying costs

If the solution to the system of FOCs (20) or (C.7) in the Appendix yields a unique and interior Nash equilibrium $\mathbf{T}^*$ and if the market is covered, i.e., if $D_0^*(\mathbf{T}^*, \boldsymbol{\tau}) = 1 - D_1^*(\mathbf{T}^*, \boldsymbol{\tau}) \in]0, 1[$, we can use these FOCs to establish the conditions under which the equilibrium demands and carriers' marginal time cost lead to $T_0^* \lesseqgtr T_1^*$. That is, setting $F(T_0^*) \lesseqgtr F(T_1^*)$ yields:

$$-\frac{k_0}{D_0^*(\mathbf{T}^*, \boldsymbol{\tau})} \lesseqgtr -\frac{k_1}{D_1^*(\mathbf{T}^*, \boldsymbol{\tau})} \quad \Leftrightarrow \quad -\frac{1 - D_0^*(\mathbf{T}^*, \boldsymbol{\tau})}{D_0^*(\mathbf{T}^*, \boldsymbol{\tau})} \lesseqgtr -\frac{k_1}{k_0}, \tag{B.17}$$

where, slightly abusing of notation, $D_i^*(\mathbf{T}^*, \boldsymbol{\tau})$ for $i = 0, 1$ is (13) if we set $F(T_0^*) \leq F(T_1^*)$ and (C.5) if we impose $F(T_0^*) \geq F(T_1^*)$. We can now analyze the above inequalities when $k_0, k_1 > 0$, $k_0 > 0$ and $k_1 < 0$, $k_0 < 0$ and $k_1 > 0$, and $k_0, k_1 < 0$. Rearranging (B.17) yields:

$$
\begin{aligned}
(i) \text{ if } k_0, k_1 > 0, \quad & D_0^*(\mathbf{T}^*, \boldsymbol{\tau}) \lesseqgtr \frac{k_0}{k_0 + k_1} \Rightarrow T_0^* \lesseqgtr T_1^*, \\
(ii) \text{ if } k_0 < 0, k_1 < 0, \quad & D_0^*(\mathbf{T}^*, \boldsymbol{\tau}) \lesseqgtr \frac{|k_0|}{|k_0| + |k_1|} \Rightarrow T_0^* \gtreqless T_1^*, \\
(iii) \text{ if } k_0 > 0 \text{ and } k_1 < 0, \quad & \frac{1 - D_0^*(\mathbf{T}^*, \boldsymbol{\tau})}{D_0^*(\mathbf{T}^*, \boldsymbol{\tau})} > -\frac{|k_1|}{k_0} \Rightarrow T_0^* < T_1^*, \\
(iv) \text{ if } k_0 < 0 \text{ and } k_1 > 0, \quad & \frac{1 - D_0^*(\mathbf{T}^*, \boldsymbol{\tau})}{D_0^*(\mathbf{T}^*, \boldsymbol{\tau})} > -\frac{k_1}{|k_0|} \Rightarrow T_0^* > T_1^*,
\end{aligned}
$$

where, slightly abusing of notation, $D_0^*(\mathbf{T}^*, \boldsymbol{\tau})$ denotes (13) if $F(T_0^*) \leq F(T_1^*)$ and (C.5) if $F(T_0^*) \geq F(T_1^*)$. Hence, in the cases (i) or (ii), the schedule differentiation pattern depends on whether the equilibrium demand $D_0^*(\mathbf{T}^*, \boldsymbol{\tau})$ is lower than, equal to or larger than the marginal time cost shares $k_0/(k_0 + k_1)$ or $|k_0|/(|k_0| + |k_1|)$. Cases (iii) and (iv) highlight that $k_0 > 0$ and $k_1 < 0$ necessarily leads to $T_0^* < T_1^*$ while $k_0 < 0$ and $k_1 > 0$ implies $T_0^* > T_1^*$.

B.7 Substituting the equilibrium fees in the equilibrium fares and in the covered market condition

Substituting the fees (25) in the fares (8) and rearranging yields:

$$
\begin{aligned}
p_0^{**} &= \frac{1}{9}[(5c_0 + 4c_1) - (5\omega_0 + 4\omega_1)] + \frac{18 + 2h}{9} + \frac{4}{9}\Phi(\mathbf{T}), \\
p_1^{**} &= \frac{1}{9}[(4c_0 + 5c_1) - (4\omega_0 + 5\omega_1)] + \frac{18 - 2h}{9} - \frac{4}{9}\Phi(\mathbf{T}).
\end{aligned}
\tag{B.18}
$$

Hence, in equilibrium, a lower marginal operational cost for *one* carrier at a facility induces a lower fare at *both* facilities; a higher per passenger commercial revenue at *one* facility reduces fare charged by *both* carriers at their departure facility. Substituting the fees (25) in carrier markups, $m_i^{**} = p_i^* - (c_i + \tau_i^*)$ for $i = 0, 1$ leads to:

$$
\begin{aligned}
m_0^{**} &= \frac{1}{9}[(c_1 - c_0) + (\omega_0 - \omega_1)] + \frac{9 + h}{18} + \frac{1}{9}\Phi(\mathbf{T}), \\
m_1^{**} &= \frac{1}{9}[(c_0 - c_1) + (\omega_1 - \omega_0)] + \frac{9 - h}{18} - \frac{1}{9}\Phi(\mathbf{T}).
\end{aligned}
\tag{B.19}
$$

Thus, a lower marginal operational cost for *one* carrier induces a higher markup for that carrier and a lower markup for the rival carrier serving the other facility. Similarly, a higher per passenger commercial revenue at *one* facility induces a higher markup for its carrier and a lower markup for the rival carrier serving the other facility. Furthermore, by using (B.19), we can rewrite (10) in terms of commercial revenues as:

$$m_0^{**} - m_1^{**} > 0 \quad \textit{iff} \quad \Delta c > -\frac{h}{2} - \Phi(\mathbf{T}) - \Delta\omega, \tag{B.20}$$

where $\Delta c = c_1 - c_0$ and $\Delta\omega = \omega_0 - \omega_1$. Recall that c_i and ω_i for $i = 0, 1$ are $\widehat{c}_i$ and $\widehat{\omega}_i$ divided by $\theta(1-h)$.

Moreover, when the marginal time cost of the carriers is null ($\widehat{k}_0 = \widehat{k}_1 = 0$) or when the departure times are given to the carriers, we can substitute the fares (B.18) in (2) and express *explicitly* the covered market condition in terms of the exogenous parameters of the model. Recalling that $\widehat{p}_i^{**} = \theta(1-h)p_i^{**}$, note first that:

$$\widehat{p}_1^{**} - \widehat{p}_0^{**} = \frac{1}{9}[(\widehat{c}_1 - \widehat{c}_0) + (\widehat{\omega}_0 - \widehat{\omega}_1) - 8\widehat{\Phi}(\mathbf{T}) - 4h\theta(1-h)]. \tag{B.21}$$

Substituting (B.21) in (2), the covered market condition (2) becomes:

$$\begin{aligned} & -\frac{\theta}{2}(1-h)(9+h) < \\ & \Delta\widehat{c} + \Delta\widehat{\omega} - 8\widehat{\Phi}(\mathbf{T}) - 9\widehat{\beta}(T_1 - T_0) \leq \Delta\widehat{c} + \Delta\widehat{\omega} - 8\widehat{\Phi}(\mathbf{T}) + 9\widehat{\gamma}(T_1 - T_0) \\ & < \frac{\theta}{2}(1-h)(9-h), \end{aligned} \tag{B.22}$$

with $\Delta\widehat{c} = \widehat{c}_1 - \widehat{c}_0$, $\Delta\widehat{\omega} = \widehat{\omega}_0 - \widehat{\omega}_1$. Further replacing $\mathbf{T}$ by $\mathbf{T}^*$, one can get rid of the endogenous departure times. Sticking to the exogenous departure times' case, we notice that the market is more likely to be covered when θ is large, h, $\widehat{\beta}$, $\widehat{\gamma}$ are low, $\Delta\widehat{c} \approx -\Delta\widehat{\omega}$ and when $\widehat{\Phi}(\mathbf{T}) \approx 0$.

B.8 Social time cost minimization (29)

Using the first derivatives (31), we deduce the following second and cross-partial derivatives:

$$\frac{\partial^2 \widehat{SC}^S(\mathbf{T})}{\partial T_0^2} = (\widehat{\beta} + \widehat{\gamma})\rho(T_0) - \frac{\widehat{\beta}^2}{\widehat{\beta} + \widehat{\gamma}}\rho(\tilde{t}), \quad \frac{\partial^2 \widehat{SC}^S(\mathbf{T})}{\partial T_1^2} = (\widehat{\beta} + \widehat{\gamma})\rho(T_1) - \frac{\widehat{\gamma}^2}{\widehat{\beta} + \widehat{\gamma}}\rho(\tilde{t}),$$

$$\frac{\partial^2 \widehat{SC}^S(\mathbf{T})}{\partial T_i \partial T_{-i}} = -\frac{\widehat{\beta}\widehat{\gamma}}{\widehat{\beta} + \widehat{\gamma}}\rho(\tilde{t}) \text{ for } i = 0, 1.$$

Setting the second derivatives larger than 0 and rearranging yields:

$$\frac{\partial^2 \widehat{SC}^S(\mathbf{T})}{\partial T_0^2} = \frac{(\widehat{\beta} + \widehat{\gamma})^2}{\widehat{\beta}^2} > \frac{\rho(\tilde{t})}{\rho(T_0)}, \quad \frac{\partial^2 \widehat{SC}^S(\mathbf{T})}{\partial T_1^2} = \frac{(\widehat{\beta} + \widehat{\gamma})^2}{\widehat{\gamma}^2} > \frac{\rho(\tilde{t})}{\rho(T_1)}.$$

Minimization requires the determinant of the Hessian matrix to be positive, i.e.,

$$2\widehat{\beta}\widehat{\gamma}\rho(T_0)\rho(T_1) + \widehat{\beta}^2\rho(T_1)[\rho(T_0) - \rho(\tilde{t})] + \widehat{\gamma}^2\rho(T_0)[\rho(T_1) - \rho(\tilde{t})] > 0.$$

From the above, we deduce that if $\rho(t) \sim \mathcal{U}[0, 24]$, then $\rho(t) = 1/24, \forall t \in [0, 24]$ and (T_0^S, T_1^S) in (32) is a unique minimum over the feasible times.

C Derivations assuming $T_0 \geq T_1$

In this Appendix, we denote the counterpart of the expressions derived assuming $T_0 \leq T_1$ with the superscript 'sym'.

C.1 Counterpart of demands (3)

When $T_0 \geq T_1$, $D_0^{\ell,sym}$ and $D_0^{r,sym}$ are given by replacing the shares m_ℓ in (B.2) and m_r in (B.6) by $m_\ell^{sym} = \int_0^{T_1} \rho(t)\mathrm{d}t$ and $m_r^{sym} = \int_{T_0}^{24} \rho(t)\mathrm{d}t$, respectively. Regarding $D_0^{c,sym}$, applying the same reasoning as in Appendix B.1, we get:

$$\begin{aligned}
\widehat{U}_0^c &= \widehat{U} - \widehat{p}_0 - \frac{\theta}{2}(x-h)^2 - \widehat{\gamma}(T_0 - t), \\
\widehat{U}_1^c &= \widehat{U} - \widehat{p}_1 - \frac{\theta}{2}(1-x)^2 - \widehat{\beta}(t - T_1), \\
\tilde{x}^{c,sym}(t) &= \frac{1}{\theta(1-h)}(\widehat{p}_1 - \widehat{p}_0) - \frac{(\widehat{\beta}T_1 + \widehat{\gamma}T_0) - (\widehat{\beta} + \widehat{\gamma})t}{\theta(1-h)} + \frac{(1+h)}{2}; \\
D_0^{c,sym} &= \left[p_1 - p_0 - (\beta T_1 + \gamma T_0) + \frac{1+h}{2}\right] m_c^{sym} + (\beta + \gamma)\bar{t}_c^{sym}, \qquad \text{(C.1)} \\
D_1^{c,sym} &= \left[p_0 - p_1 + (\beta T_1 + \gamma T_0) + \frac{1-h}{2}\right] m_c^{sym} - (\beta + \gamma)\bar{t}_c^{sym},
\end{aligned}$$

where $m_c^{sym} = \int_{T_1}^{T_0} \rho(t)\mathrm{d}t$ and $\bar{t}_c^{sym} = \int_{T_1}^{T_0} t\,\rho(t)\mathrm{d}t$. Given that $D_0^{sym} = D_0^{l,sym} + D_0^{c,sym} + D_0^{r,sym}$ and $D_1^{sym} = 1 - D_1^{sym}$, we have the following aggregate demands:

$$\begin{aligned}
D_0^{sym} &= p_1 - p_0 + \frac{1+h}{2} + \Phi^{sym}(\mathbf{T}), \\
D_1^{sym} &= p_0 - p_1 + \frac{1-h}{2} - \Phi^{sym}(\mathbf{T}),
\end{aligned} \qquad \text{(C.2)}$$

where the counterpart of the SDC difference (4) is given by:

$$\Phi^{sym}(\mathbf{T}) = \gamma(T_1 - T_0)m_l^{sym} - (\beta T_1 + \gamma T_0)m_c^{sym} - \beta(T_1 - T_0)m_r^{sym} + (\beta + \gamma)\bar{t}_c^{sym}, \quad \text{(C.3)}$$

where $m_l^{sym} + m_c^{sym} + m_r^{sym} = 1$.

C.2 Counterpart of the covered market condition (2)

Following the same steps as in Appendix B.2, the covered market condition (2) is:

$$-\frac{\theta}{2}(1-h^2) < \widehat{p}_1 - \widehat{p}_0 - \widehat{\gamma}(T_0 - T_1) \leq \widehat{p}_1 - \widehat{p}_0 + \widehat{\beta}(T_0 - T_1) < \frac{\theta}{2}(1-h)^2. \tag{C.4}$$

C.3 Proof of Lemma 2 when $T_0 \geq T_1$

Consider $\Phi^{sym}(\mathbf{T})$ in Eq.(C.3). To compute $\partial\Phi^{sym}(\mathbf{T})/\partial T_i$ for $i = 0, 1$, use terms m_l^{sym}, m_c^{sym}, m_r^{sym} and $\bar{t}_c^{sym}$ defined in Appendix C.1. Note first that:

$$\frac{\partial m_l^{sym}}{\partial T_0} = 0, \qquad \frac{\partial m_c^{sym}}{\partial T_0} = \rho(T_0), \qquad \frac{\partial m_r^{sym}}{\partial T_0} = -\rho(T_0), \qquad \frac{\partial \bar{t}_c^{sym}}{\partial T_0} = T_0\rho(T_0),$$
$$\frac{\partial m_l^{sym}}{\partial T_1} = \rho(T_1), \qquad \frac{\partial m_c^{sym}}{\partial T_1} = -\rho(T_1), \qquad \frac{\partial m_r^{sym}}{\partial T_1} = 0, \qquad \frac{\partial \bar{t}_c^{sym}}{\partial T_1} = -T_1\rho(T_1).$$

Using the above, we obtain:

$$\frac{\partial\Phi^{sym}(\mathbf{T})}{\partial T_0} = (\beta+\gamma)m_r^{sym} - \gamma, \qquad \frac{\partial\Phi^{sym}(\mathbf{T})}{\partial T_1} = (\beta+\gamma)m_\ell^{sym} - \beta,$$
$$\frac{\partial^2\Phi^{sym}(\mathbf{T})}{\partial T_0^2} = -(\beta+\gamma)\rho(T_0), \qquad \frac{\partial^2\Phi^{sym}(\mathbf{T})}{\partial T_1^2} = (\beta+\gamma)\rho(T_1).$$

where we use $m_\ell^{sym} + m_c^{sym} = 1 - m_r^{sym}$ to get $\partial\Phi^{sym}(\mathbf{T})/\partial T_0$ and $m_c^{sym} + m_r^{sym} = 1 - m_\ell^{sym}$ to get $\partial\Phi^{sym}(\mathbf{T})/\partial T_1$. Again, setting each first derivative equal to zero leads to $T_0^* = T_1^* = F^{-1}[\beta/(\beta+\gamma)]$.

C.4 Counterpart of the time game of Section 2.2.2

When $T_0 \geq T_1$, by Proposition 2, the counterpart of the equilibrium demands (13) are given by:

$$\begin{aligned} D_0^{sym,*}(\mathbf{T},\boldsymbol{\tau}) &= \frac{1}{6}[3 + h + 2\Delta\widetilde{c} + 2\Phi^{sym}(\mathbf{T})], \\ D_1^{sym,*}(\mathbf{T},\boldsymbol{\tau}) &= \frac{1}{6}[3 - h - 2\Delta\widetilde{c} - 2\Phi^{sym}(\mathbf{T})], \end{aligned} \tag{C.5}$$

and FOCs (19) become:

$$\frac{\partial\pi_0^{sym,*}(\mathbf{T},\boldsymbol{\tau})}{\partial T_0} = \frac{2}{3}\ [(\beta+\gamma)\ m_r^{sym} - \gamma]\ D_0^{sym,*}(\mathbf{T},\boldsymbol{\tau}) - \frac{\partial K(T_0)}{\partial T_0} = 0,$$
$$\frac{\partial\pi_1^{sym,*}(\mathbf{T},\boldsymbol{\tau})}{\partial T_1} = -\frac{2}{3}\ [(\beta+\gamma)\ m_l^{sym} - \beta]\ D_1^{sym,*}(\mathbf{T},\boldsymbol{\tau}) - \frac{\partial K(T_1)}{\partial T_1} = 0,$$

where m_r^{sym} and m_ℓ^{sym}, defined in Appendix C.1, denote the share of travellers with desired times earlier than T_1 and later than T_0, respectively. Consider the case where $\partial K(T_i)/\partial T_i = k_i = 0$ for $i = 0, 1$. Solving the above system of FOCs with respect to T_i yields the counterpart of the equilibrium departure times (22):

$$F(T_0) = F(T_1) = \frac{\beta}{\beta+\gamma} \Rightarrow T^{sym,*}|_{k_0=k_1=0} = F^{-1}\left(\frac{\beta}{\beta+\gamma}\right). \tag{C.6}$$

Turning to the case where $k_0, k_1 \neq 0$ in (18), the counterpart of (20) is:

$$F(T_i) = \frac{\beta}{\beta+\gamma} - \frac{3k_i}{2(\beta+\gamma)D_i^{sym,*}(\mathbf{T}, \boldsymbol{\tau})}, \quad i = 0, 1. \tag{C.7}$$

Assuming that a unique and interior solution exists for each T_i in (C.7), profit maximization requires the SOCs to hold, that is:

$$k_i^2 < \frac{4}{3}\rho(T_i^{sym,*})(\beta+\gamma)D_i^{sym,*3}(\mathbf{T}^{sym,*}, \boldsymbol{\tau}).$$

C.5 Counterpart of the social time cost minimization (29)

When $T_0 \geq T_1$, the $\widehat{SDC}^S(\mathbf{T})$ term (29) becomes:

$$\begin{aligned}\widehat{SDC}^{sym,S}(\mathbf{T}) = &\int_0^{T_1} \widehat{\gamma}(T_1 - t)\rho(t)\,\mathrm{d}t + \int_{T_1}^{\widetilde{t}^{sym}} \widehat{\beta}(t - T_1)\rho(t)\,\mathrm{d}t + \\ &\int_{\widetilde{t}^{sym}}^{T_0} \widehat{\gamma}(T_0 - t)\rho(t)\,\mathrm{d}t + \int_{T_0}^{24} \widehat{\beta}(t - T_0)\rho(t)\,\mathrm{d}t,\end{aligned}$$

where $\widetilde{t}^{sym} = (\widehat{\gamma}T_0 + \widehat{\beta}T_1)/(\widehat{\beta} + \widehat{\gamma})$. Then, the counterpart of FOCs (29) are:

$$\begin{aligned}\frac{\partial \widehat{SC}^{sym,S}(\mathbf{T})}{\partial T_0} &= \widehat{\gamma}\widetilde{m}_r^{sym} - \widehat{\beta}m_r^{sym} + \widehat{k}_0 = 0, \\ \frac{\partial \widehat{SC}^{sym,S}(\mathbf{T})}{\partial T_1} &= \widehat{\gamma}m_\ell^{sym} - \widehat{\beta}\widetilde{m}_\ell^{sym} + \widehat{k}_1 = 0,\end{aligned} \tag{C.8}$$

where $\widetilde{m}_r^{sym} = \int_{\widetilde{t}^{sym}}^{T_0} \rho(t)dt$, $\widetilde{m}_\ell^{sym} = \int_{T_1}^{\widetilde{t}^{sym}} \rho(t)dt$ and where m_r^{sym} and m_ℓ^{sym} are given in Appendix C.1. Using (C.8), we deduce the following second and cross-partial derivatives:

$$\frac{\partial^2 \widehat{SC}^S(\mathbf{T})}{\partial T_0^2} = (\widehat{\beta} + \widehat{\gamma})\rho(T_0) - \frac{\widehat{\gamma}^2}{\widehat{\beta} + \widehat{\gamma}}\rho(\widetilde{t}^{sym}), \quad \frac{\partial^2 \widehat{SC}^S(\mathbf{T})}{\partial T_1^2} = (\widehat{\beta} + \widehat{\gamma})\rho(T_1) - \frac{\widehat{\beta}^2}{\widehat{\beta} + \widehat{\gamma}}\rho(\widetilde{t}^{sym}),$$

$$\frac{\partial^2 \widehat{SC}^{sym,S}(\mathbf{T})}{\partial T_i \partial T_{-i}} = -\frac{\widehat{\beta}\widehat{\gamma}}{\widehat{\beta} + \widehat{\gamma}}\rho(\widetilde{t}^{sym}), \quad i = 0, 1.$$

Setting the second derivatives larger than 0 and rearranging yields:

$$\frac{\partial^2 \widehat{SC}^{sym,S}(\mathbf{T})}{\partial T_0^2} = \frac{(\widehat{\beta}+\widehat{\gamma})^2}{\widehat{\gamma}^2} > \frac{\rho(\tilde{t}^{sym})}{\rho(T_0)}, \quad \frac{\partial^2 \widehat{SC}^{sym,S}(\mathbf{T})}{\partial T_1^2} = \frac{(\widehat{\beta}+\widehat{\gamma})^2}{\widehat{\beta}^2} > \frac{\rho(\tilde{t}^{sym})}{\rho(T_1)}.$$

Minimization requires the determinant of the Hessian matrix to be positive, i.e.,

$$2\widehat{\beta}\widehat{\gamma}\rho(T_0)\rho(T_1) + \widehat{\beta}^2\rho(T_0)[\rho(T_1) - \rho(\tilde{t}^{sym})] + \widehat{\gamma}^2\rho(T_1)[\rho(T_0) - \rho(\tilde{t}^{sym})] > 0.$$

When $\rho(t) \sim \mathcal{U}[0, 24]$, then $\rho(t) = 1/24, \forall t \in [0, 24]$ and the sufficient condition for (global) minimum holds. With the latter distribution, the counterpart of the SC^S function in (29) is:

$$\widehat{SC}^{sym,S} = \frac{\widehat{\beta}(\widehat{\beta}+2\widehat{\gamma})T_0^2 - 48\widehat{\beta}(\widehat{\beta}+\widehat{\gamma})T_0 + \widehat{\gamma}(2\widehat{\beta}+\widehat{\gamma})T_1^2 - 2\widehat{\beta}\widehat{\gamma}T_0T_1 + 576\widehat{\beta}(\widehat{\beta}+\widehat{\gamma})}{48(\widehat{\beta}+\widehat{\gamma})} + \widehat{K}^S(\mathbf{T}).$$

Solving the FOCs yields the socially optimal departure times:

$$T_i^{sym,S}\Big|_{\widehat{k}_0,\widehat{k}_1\neq 0} = T_i^{sym,S}\Big|_{\widehat{k}_0=\widehat{k}_1=0} - a_i^{sym}\widehat{k}_0 - b_i^{sym}\widehat{k}_1, \quad i = 0, 1,$$

where $T_0^{sym,S}\Big|_{\widehat{k}_0=\widehat{k}_1=0} = 12\left[1 + \frac{\widehat{\beta}}{\widehat{\beta}+\widehat{\gamma}}\right]$, $T_1^{sym,S}\Big|_{\widehat{k}_0=\widehat{k}_1=0} = \frac{12\widehat{\beta}}{\widehat{\beta}+\widehat{\gamma}}$, $a_0^{sym} = \frac{12(2\widehat{\beta}+\widehat{\gamma})}{\widehat{\beta}(\widehat{\beta}+\widehat{\gamma})} > 0$, $b_0^{sym} = a_1^{sym} = \frac{12}{\widehat{\beta}+\widehat{\gamma}} > 0$ and $b_1^{sym} = \frac{12(\widehat{\beta}+2\widehat{\gamma})}{\widehat{\gamma}(\widehat{\beta}+\widehat{\gamma})} > 0$. Again, setting $\widehat{k}_0 = \widehat{k}_1 = 0$ simplifies the analysis. Then, by using (C.6) and the uniform distribution, we get:

$$\Delta T_0^{sym} = T_0^{sym,S}\Big|_{\widehat{k}_0=\widehat{k}_1=0} - T^{sym,*}|_{\widehat{k}_0=\widehat{k}_1=0} = 12\frac{\widehat{\gamma}}{\widehat{\beta}+\widehat{\gamma}},$$
$$\Delta T_1^{sym} = T_1^{sym,S}\Big|_{\widehat{k}_0=\widehat{k}_1=0} - T^{sym,*}|_{\widehat{k}_0=\widehat{k}_1=0} = -12\frac{\widehat{\beta}}{\widehat{\beta}+\widehat{\gamma}}.$$

Using $T_i^{sym,S}\Big|_{\widehat{k}_0=\widehat{k}_1=0}$ in (C.7), we get an explicit pricing rule for the departure times that holds with linear costs (18), i.e.,

$$\widehat{k}_0^{sym,S} = -\frac{\widehat{\gamma}D_0^{sym,S}}{3}, \quad \widehat{k}_1^{sym,S} = \frac{\widehat{\beta}D_1^{sym,S}}{3}.$$

where $D_i^{sym,S} \equiv D_i^{sym,*}(\mathbf{T}^{sym,S}, \boldsymbol{\tau})$ and $\mathbf{T}^{sym,S} \equiv (T_0^{sym,S}\Big|_{\widehat{k}_0=\widehat{k}_1=0}, T_1^{sym,S}\Big|_{\widehat{k}_0=\widehat{k}_1=0})$. The schedule regulator should set a decreasing cost in the time of day for the carrier serving facility 0 to get the late socially optimal service time and an increasing one for the carrier operating at facility 1 to get the early socially optimal departure time.